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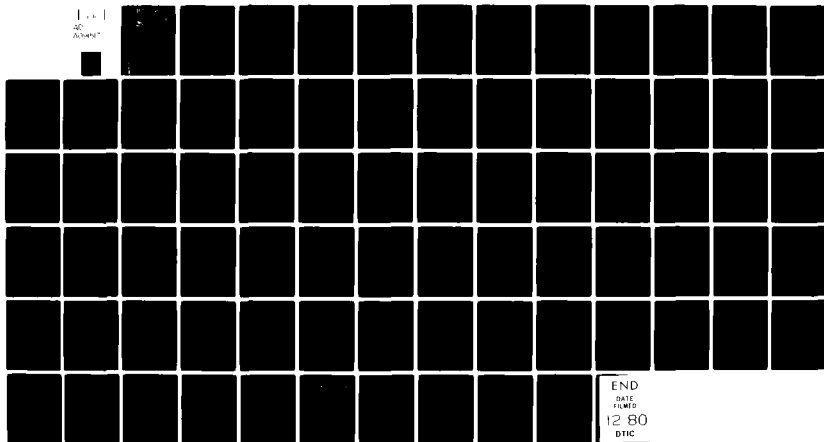   DESIGN CRITERIA FOR DEFLECTION CAPACITY OF CONVENTIONALLY REINF--ETC(U)
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CIVIL ENGINEERING LABORATORY
Naval Construction Battalion Center
Port Hueneme, CA

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DESIGN CRITERIA FOR DEFLECTION CAPACITY OF
CONVENTIONALLY REINFORCED CONCRETE SLABS,
PHASE II - DESIGN AND CONSTRUCTION REQUIREMENTS

October 1980

An Investigation Conducted by
CONSTRUCTION TECHNOLOGY LABORATORIES
Structural Analytical Section
5420 Old Orchard Road
Skokie, Illinois 60077

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Incipient collapse deflection data and an analysis method, reported in Phase I of this study, were used to propose an incipient collapse deflection criteria for one-way and two-way slabs. A relationship for the incipient collapse deflection of slabs, when tensile membrane action can be developed, is given in terms of the short span and steel rupture strains. Structural design details, to insure that membrane action can be developed, are discussed and experimental verification tests are recommended. The Phase III report will recommend a design procedure for one-way and two-way slabs subjected to blast loads.

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OF CONVENTIONALLY REINFORCED
CONCRETE SLABS

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Construction Requirements

by
T. Takayanagi, A.T. Derecho, and M. Iqbal*

1. INTRODUCTION

1.1 Objective and Scope

The primary objective of this investigation is to develop design
criteria for conventionally reinforced concrete slabs under
static uniform load based on incipient collapse conditions.
Major emphasis is placed on the deflection capacity associated
with incipient collapse. This involves a reexamination of rele-
vant design criteria contained in NAVFAC P-397, "Structures to
Resist the Effects of Accidental Explosions," (1) in the light
of experimental and analytical data that have become available
since the publication of the manual in 1969.

The investigation has been subdivided into three phases. Under
Phase I (2), a definition of incipient collapse for convention-
ally reinforced concrete slabs was proposed. Then, a review of
experimental and analytical work on one-way and two-way reinfor-
ced concrete slabs was presented. Slabs with and without
lateral and rotational edge restraint were considered. Parti-
cular emphasis was placed on investigations considering tensile
membrane action. Available methods for estimating incipient
collapse deflection of reinforced concrete slabs were examined.
Finally, a method for estimating slab deflection capacity was
proposed.

The review of experimental data in Phase I dealt mainly with
slabs supported on beams along four edges or continuous over
supporting beams. Practically no mention was made of flat
slabs, i.e, slabs with drop panels. The reason for this is that
there are no reported tests of flat slabs that have been carried
out to the point of incipient collapse. In this type of struc-
ture, the major concern lies in the relatively high shearing
stresses that occur along the periphery of the column support.
Failure generally occurs by a punching failure of the slab at
the column support. The few tests that have been carried out

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have concentrated on the punching shear problem, which represents the most critical consideration associated with this type of structure.

Phase II of the investigation was originally planned to consist of a parametric study of a number of variables, using an analytical procedure to be recommended in Phase I. The purpose of the study would have been to identify the most significant parameters in terms of their effects on incipient collapse deflection capacity of slabs. Design criteria reflecting the influence of the major parameters were to be developed. However, the lack of a suitable analytical tool with which to examine the response of slabs of varying configuration in the range approaching failure precluded undertaking the parametric investigation. Specifically, no available procedure accounts for the changes in stresses and deformations in different parts of a slab under increasing load. Such a method must necessarily consider the changing properties of portions of the slab as cracking occurs and as crack patterns change. It must also consider the in-plane resistance and deformation of the slab in addition to flexural action. Furthermore, the method should preferably include effects of geometric nonlinearities. The latter effects may be important in view of the significant deflections involved in the tensile membrane action range.

Because of the lack of a suitable analytical tool, reliance had to be placed on the meager experimental data available to determine the major parameters affecting incipient collapse deflection. Also scheduled for Phase II is the development of design and construction requirements to ensure attainment of the predicted incipient collapse deflection.

A major conclusion of the Phase I study is that the best available method of estimating the incipient collapse deflection of conventionally reinforced slabs under static loading is given by an expression based on the assumption of pure membrane action in the slab. An expression for the collapse deflection based on this assumption was proposed in the Phase I report. The recommended relationship gives incipient collapse deflection, δ_{ult}, as a function of two parameters: the short span, L_y, and the rupture strain of the reinforcement, ε_u.

The proposed relationship implies that on the basis of available experimental data, the two most important variables affecting incipient collapse deflection capacity of conventionally reinforced slabs are length of the short span and ultimate strain of the reinforcement.

The proposed expression includes a factor the value of which may be selected to yield a specific probability that the incipient collapse deflection of a given slab will exceed the predicted deflection.

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Since the major parameters affecting incipient collapse deflection capacity were identified and the appropriate relationship presented in Phase I, work under Phase II proceeded with the development of design and construction requirements. These requirements are intended to ensure attainment of maximum deflection prior to collapse.

Work accomplished in Phase II of the investigation is presented in this report. By way of recapitulation, a brief description of the slab load-versus-deflection relationship is presented. The proposed expression for predicting deflection at incipient collapse is restated and comparison between the prediction line and experimental data indicated. Finally, design and construction requirements necessary to develop tensile membrane behavior at incipient collapse are developed.

The work under Phase III will summarize the work under Phases I and II in the form of a supplement to NAVFAC P-397.

The scope of this investigation is limited to one-way and two-way slabs under uniformly distributed static load near incipient collapse.)

1.2 Load-Deflection Relationship and Incipient Collapse

As examined in the report on Phase I (2), the load-deflection relationship of uniformly loaded reinforced concrete slabs is significantly influenced by the boundary conditions along the slab edges. This is illustrated in Fig. 1.

1.2.1 Simply-Supported Slabs.The solid curve in the figure shows that a two-way, simply-supported slab deflects elastically and then elasto-plastically as the load is increased from A to B. Near load stage B, a yield-line pattern develops and the slab deflects at a faster rate. Beyond this stage, the slab is continuously stretched at the center with cracks penetrating through the entire slab thickness. The center portion of the slab acts essentially as a tensile membrane. Tensile forces at the center require existence of a compressive ring capable of resisting radial tension. Compression is often taken by edge beams, but if beams are absent, the slab will naturally develop a compressive ring, as shown in Fig. 2.

In the tensile membrane regime, depth of the compressive stress block in the yield-line near the corners is greatly increased, while the tensile cracks at the center may go completely through the slab. The increase in deflection gives rise to an increasing elongation that ultimately leads to fracture of the reinforcement.

1.2.2 Restrained Slabs. When slab edges are restrained against lateral movement, slab capacity is enhanced in the early stages of loading due to arching (compressive membrane)

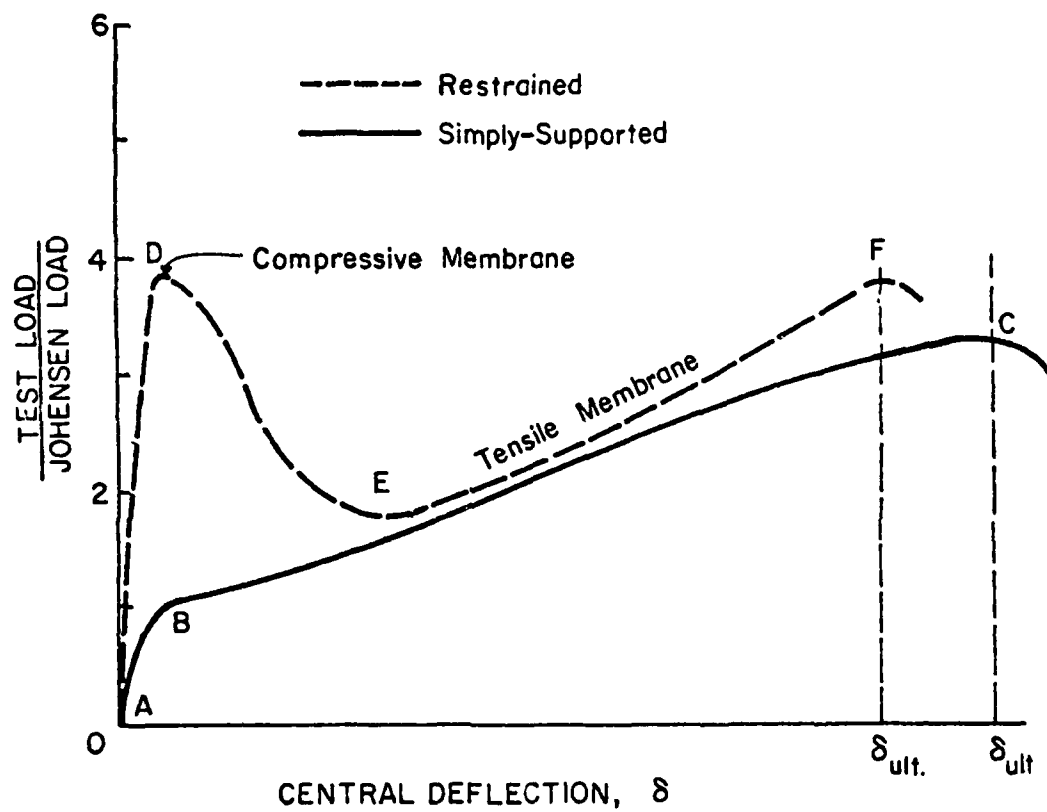Fig. 1 Load-versus-Deflection Relationship for
Two-Way Reinforced Concrete Slabs

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Outer part of Slab
Functions as Compression
Ring

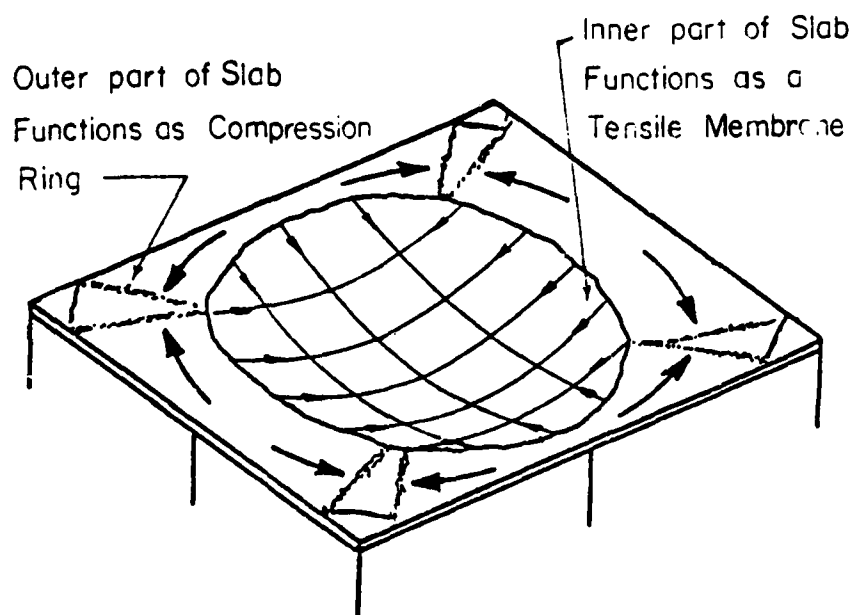Inner part of Slab
Functions as a
Tensile Membrane

Fig. 2 Behavior of a Simply-Supported Slab Near
Incipient Collapse Deflection

action, as shown by the dashed curve in Fig. 1. This restraint reaches a maximum at Point D when crushing of the concrete in compression occurs. Immediately beyond D, the load carried by the slab decreases rapidly. This is sometimes referred to as the "snap through" phase.

After the concrete crushes and as Point E is approached, membrane action in the central region of the slab changes from compressive to tensile. Beyond E, the slab carries load by the reinforcement acting as a plastic tensile membrane, with cracks penetrating the slab thickness. The slab continues to carry greater load with increasing deflection until the reinforcement ruptures at F. Point F (or Point C for simply-supported slabs) in Fig. 1 corresponds to the condition of incipient collapse.

In both simply-supported and restrained slabs, rupture of the reinforcement precipitates collapse. Alternatively, shear failure or failure of bond between reinforcement and concrete may trigger premature collapse.

The term incipient collapse for conventionally reinforced concrete slabs is defined as that state of a slab characterized by a drop in the load capacity following mobilization of tensile membrane action. The collapse condition is associated with tensile rupture of the flexural reinforcement. It is assumed that the slab is properly designed to preclude premature bond or shear failure. It is further assumed that the concrete is effectively confined within the reinforcing mesh so that no major gaps occur in the slab as a result of concrete fragments falling off.

2. DESIGN CRITERIA FOR INCIPIENT COLLAPSE DEFLECTION

In the following sections, the equation for incipient collapse deflection proposed in the Phase I Report is restated. Then, probability concepts associated with test results on ultimate deflection and steel rupture strain are introduced in an evaluation of the proposed equation. Finally, design criteria for incipient collapse deflection are proposed.

2.1 Equation for Incipient Collapse Deflection

It was stated in the Phase I Report (2) that incipient collapse deflection of conventionally reinforced concrete slabs can be estimated using the expression

$$\delta_{ult} = k \; L_y \; \sqrt{\epsilon_u} \qquad (1)$$

where
L_y = short span of slab

ϵ_u = breaking strain of flexural reinforcement

k = a factor to account for uncertainties

The basic form of Eq. 1 was derived on the assumption that a representative slab strip takes the form of a parabolic cable in the tensile membrane regime. The value of k is required to account for uncertainties such as the disparity between the deflection associated with pure cable action and actual slab behavior. A major difference between the two is the non-uniform strain distribution along the length of the reinforcement associated with cracking in the slab.

Two parameters are needed to estimate slab incipient collapse deflection, δ_{ult}. These are slab short span, L_y, and breaking strain of flexural reinforcement, ϵ_u. Figure 3 shows the proposed equation, with k assigned a value of 0.25, compared with available experimental data on restrained two-way slabs. The plotted test data represent only those tests in which incipient collapse was observed and ultimate strain of the steel reinforcement at rupture was known. The geometric and material properties of these test specimens are summarized in Table 1 along with relevant test results.

2.2 Probability of Exceedance of δ_{ult} – Steel Rupture Strain, ϵ_u, Specified.

Although available experimental data is meager, the prediction of incipient collapse deflection using Eq. 1 may be treated within a probabilistic framework if the appropriate probability density functions can be assumed as adequately defined. The assumed density functions can be refined as more data become available. In the following, two alternative approaches are
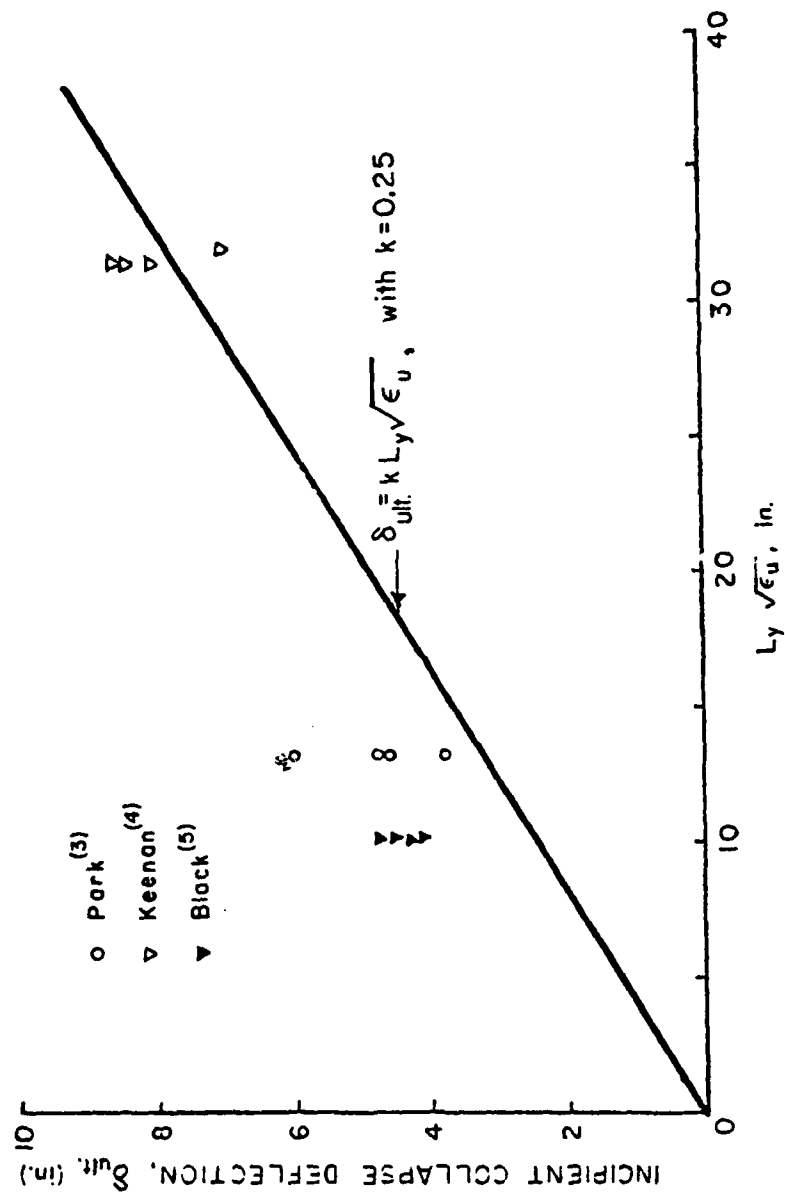
Fig. 3 Incipient Collapse Deflection of Conventionally Reinforced
Two-Way Slabs as a Function of Short Span, L_y, and Steel
Breaking Strain, ε_u

Table 1 Properties and Test Results of Two-Way Restrained Slabs

Investigators**	Mark	Geometric and Material Properties									Test Results			
		Clear Slab Dimension L_y x L_x x h in.	L_y/L_x	L_x/h	Steel Reinforcement, %				f_y ksi	ϵ_u	δ_t in.	δ_t/L_y	Maximum Edge Rotation degrees	Remarks
					Short Span		Long Span							
					Top	Bot.	Top	Bot.						
Park[3]	A1	40x60x2	1.5	20	0.38	0.19	0.41	0.20	45.5	0.11	3.8	0.10	11	Rectangular; orthotropic; edges fully restrained
	A2				0.84	0.42	0.43	0.21			4.8	0.12	14	
	A3				1.44	0.72	0.45	0.22			6.0	0.15	17	
	A4				2.42	1.21	0.47	0.23			4.6	0.12	13	
Keenan[4]	3S1	72x72x3	1.0	24	0.82	0.82	0.82	0.82	49.6	0.19	8.4	0.12	13	Fully restrained
	3S3				0.82	0.82	0.82	0.82	49.6	0.19	8.5	0.12	13	
	3S4				0.82	0.82	0.82	0.82	49.6	0.19	8.0	0.11	13	
	4.75S1	72x72x4.75	1.0	15.2	0.89	0.89	0.89	0.89	47.4	0.20	7.0	0.10	11	
Black[5]	1S1	29x29x0.89	1.0	33	0	0.87	0	0.87	43.6	0.12*	4.47	0.15	17	Fully restrained
	1S2				0	0.87	0	0.87	43.6		4.1	0.14	16	
	1S3				0	0.87	0	0.87	43.6		4.64	0.16	18	
	1S4				0	1.17	0	1.17	43.6		4.1	0.14	16	

*Average value for wire tests in tension
**Only those tests in which incipient collapse was observed and rupture strain of steel bar was known

used in determining the value of the factor k corresponding to specified probabilities that the incipient collapse deflection exceeds the value predicted by Eq. 1.

In the first approach, probability density functions for the factor k are determined based on the ultimate deflection and steel rupture strain measured in tests. Where data on steel rupture strain are not available, a reasonably conservative value was assumed.

Figure 4 shows a histogram of the ratio $k = \delta_{ult}/(L_y \sqrt{\epsilon_u})$, for the twelve tests plotted in Fig. 3. Also shown are the corresponding continuous distributions of the factor k, one based on an assumed normal distribution and the other a lognormal distribution. Because of the smallness of the sample size used in preparing the histogram, the fit is not good for either assumed continuous distribution.

Based on an assumed normal distribution, the probability that incipient collapse deflection is greater than $\delta_{ult} = 0.25\ L_y\ \sqrt{\epsilon_u}$ (i.e., k = 0.25) is 0.87. The corresponding probability for the lognormal distribution is 0.89. For a probability that the incipient collapse deflection in 95% of all cases exceeds that predicted by the equation $\delta_{ult} = k\ L_y\ \sqrt{\epsilon_u}$, a value of k = 0.21 is required for an assumed normal distribution and k = 0.23 for a lognormal distribution.

For simply-supported slabs, no data on steel rupture strain are available. To obtain a value for the ratio k, a value of steel rupture strain, $\epsilon_u = 0.15$, was assumed. Only those tests in which specimens were believed to be loaded up to or close to incipient collapse were considered (6, 7). The geometric and material properties of these test specimens are summarized in Table 2.

Figure 5 shows a histogram of the factor k for the six simply-supported slabs considered. Also shown are the corresponding continuous distributions of the factor k, one based on an assumed normal distribution and the other a lognormal distribution. The normal and lognormal distributions give practically the same value of probability of exceedance in this case. The probability that incipient collapse deflection is greater than $\delta_{ult} = 0.40\ L_y\ \sqrt{\epsilon_u}$ is 88%. This k value of 0.40 compares with k = 0.25 for about the same probability in restrained slabs. The corresponding value of k for a 95% probability of exceedance is 0.38. With k = 0.25, the associated probability of exceedance is close to 100%. It is thus seen that value of k = 0.25 proposed in Phase I for restrained slabs is conservative for simply-supported slabs.

Values of k corresponding to different probabilities of exceedance, assuming a normal distribution of k, together with values
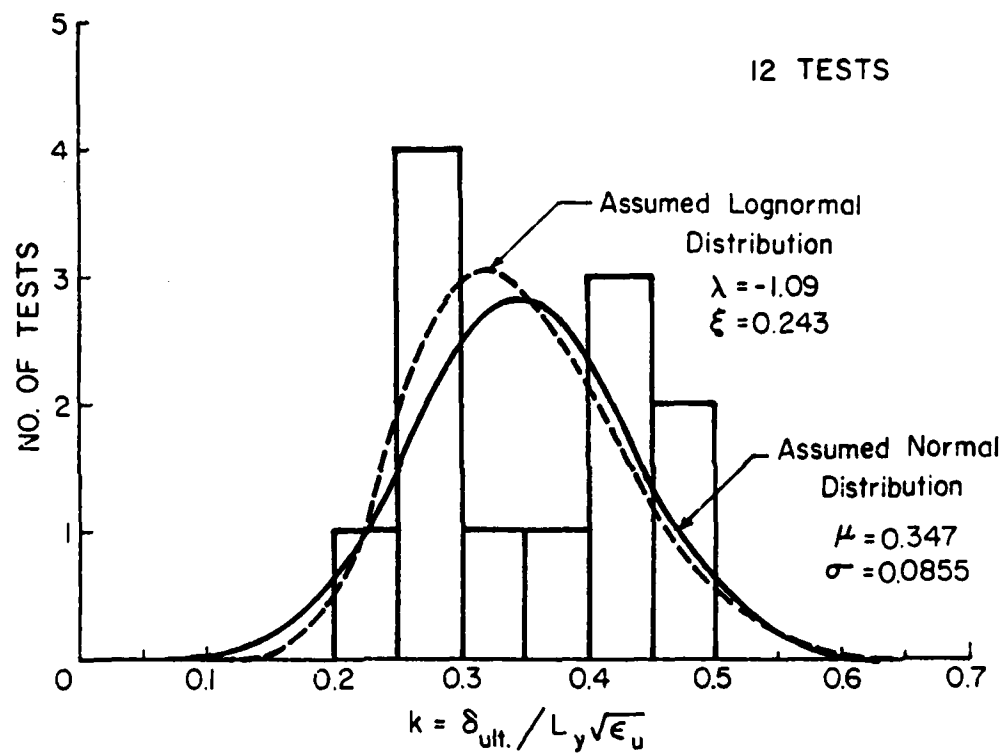
Fig. 4   Distribution of Incipient Collapse Deflection
Factor, k, for Restrained Slab Test Data

Table 2 Properties and Test Results of Two-Way Simply Supported Slabs

Investigators	Mark	Clear Slab Dimensions $L_y \times L_x \times h$ in.	L_y/L_x	L_x/h	Steel Reinforcement, %		f_y ksi	δ_t in.	δ_t/L_y	Maximum Edge Rotation degrees	Remarks
					Short Span	Long Span					
					Geometric and Material Properties			**Test Results**			
Sawczuk and Winnicki (7)	I1	43x63x1.18	1.45	37.0	0.91	0.91	38.3	7.9	0.18	20	
	I2	39x79x1.18	2.00	33.0	0.91	0.91		7.9	0.20	22	
	II1	43x63x1.18	1.45	37.0	0.45	0.45		7.9	0.18	20	
Brotchie and Holley (6)	8	15x15x0.75	1.00	20.0	1.00	1.00	60.0	2.2	0.15	16	Square Isotropic
	9			20.0	3.00	3.00		2.4	0.16	18	
	12			10.0	1.00	1.00		2.6	0.17	19	

-12-
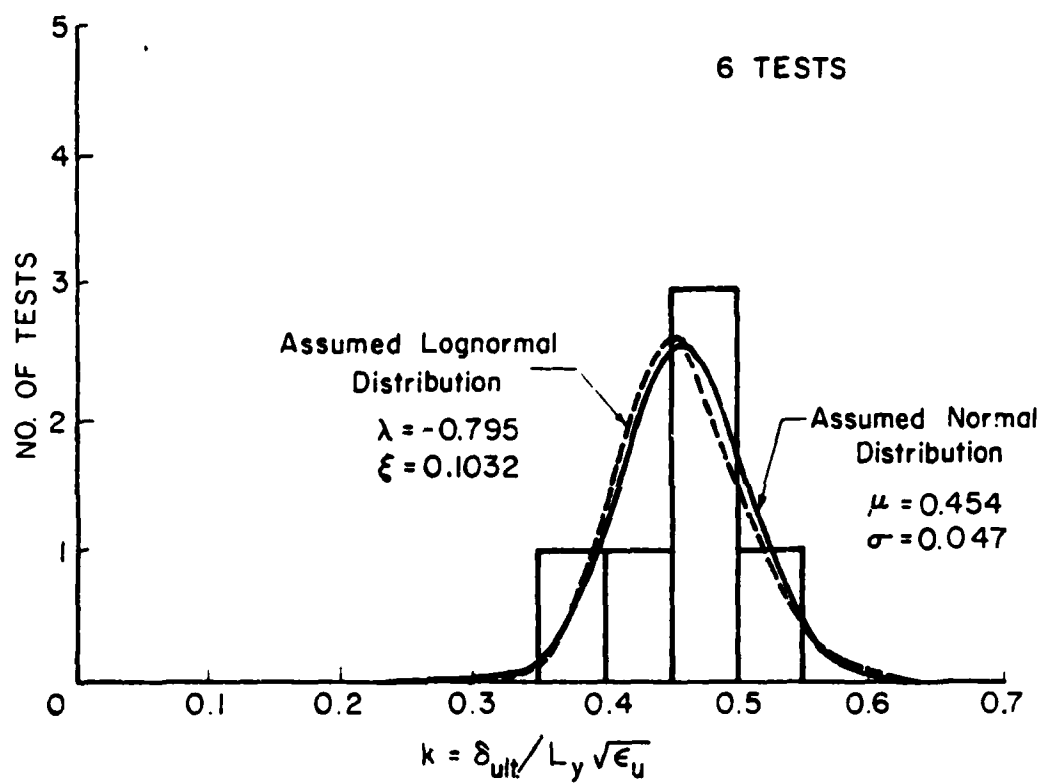
Fig. 5   Distribution of Incipient Collapse Deflection
Factor k, for Simply-Supported Slab Test Data

of the mean, μ, and standard deviation, σ, for k are summarized in Table 3.

Although Eq. 1 is based on the results of tests on small-scale specimens, it is assumed that the expression is directly applicable to full-scale slabs. Since the primary mechanism of resistance in the tensile membrane range is relatively simple and the variables considered in Eq. 1 are directly related to this simple mechanism, there is good justification for the application of Eq. 1 to full-scale slabs.

2.3 Probability of Exceedance of δ_{ult} - Steel Rupture Strain, ε_u, Considered as Independent Random Variable

The preceding discussion assumed the factor k to be a function of only one independent variable, $\delta_{ult}/(L_y \sqrt{\varepsilon_u})$. Each sample value of $\delta_{ult}/(L_y \sqrt{\varepsilon_u})$ was calculated from corresponding measured values of δ_{ult}, L_y, and ε_u for each test specimen considered.

Probability density functions of the factor k may also be derived as a function of two random variables. These are the ratio of ultimate deflection to short span, δ_{ult}/L_y, and steel rupture strain, ε_u. These two random variables have their own probability density functions.

The probability density function of the ratio δ_{ult}/L_y can be obtained from selected test results as listed in Tables 1 and 2. Because of the scarcity of data on slabs tested to incipient collapse, the probability density function for δ_{ult}/L_y will have to await further tests for a more accurate definition. There are, however, sufficient data on steel rupture strain, in addition to values related to the slab tests, to allow a much better definition of the associated probability density function. For this purpose, the results of coupon tests of reinforcing bars from various sources can be used. The probability of exceedance of δ_{ult} will be derived based on the density function for k determined using this second approach.

The basic relationships used in the derivation of the probability density function of a random variable that is a function of several random variables are given below.

If Y is a function of two random variables, that is,

$$Y = g(X_1, X_2) \tag{2}$$

The approximate mean and variance of Y are obtained as follows (8). By expanding the function $g(X_1, X_2)$ in a Taylor series about the mean values μ_{X_1} and μ_{X_2}, Eq. 2 becomes:

Table 3 Values of k for Selected Probabilities of Exceedance - Steel Rupture Strain Specified

Support Conditions	k		Values of k Corresponding to Probabilities of Exceedance		
	μ	σ	90%	95%	99%
Restrained 12 Tests	0.347	0.086	0.238	0.207	0.148
Simply-Supported* 6 Tests	0.454	0.047	0.394	0.377	0.344

*ε_u assumed equal to 0.15.

$$Y = g(\mu_{x_1}, \mu_{x_2}) + \sum_{i=1}^{2} (X_i - \mu_{x_i}) \frac{\partial g}{\partial X_i}$$

$$+ \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} (X_i - \mu_{x_i})(X_j - \mu_{x_j}) \frac{\partial^2 g}{\partial X_i \partial X_j} + .. \qquad (3)$$

where the derivatives are evaluated at μ_{x_1} and μ_{x_2}.

Truncating the series at the third terms, and assuming X_1 and X_2 to be statistically independent, the approximate mean and variance of Y can be expressed respectively as

$$E(Y) \simeq g(\mu_{x_1}, \mu_{x_2}) + \frac{1}{2} \sum_{i=1}^{2} \frac{\partial^2 g}{\partial X_i^2} \text{Var}(X_i) \qquad (4)$$

$$\text{Var}(Y) \simeq \sum_{i=1}^{2} C_i^2 \text{Var}(X_i) \qquad (5)$$

where C_i are the values of the partial derivatives $\partial g / \partial X_i$, evaluated at μ_{x_1} and μ_{x_2}.

In Eq. 1, the factor k can be considered as a function of the variables δ_{ult}/L_y and ε_u ;

$$k = \left(\frac{\delta_{ult}}{L_y}\right)\left(\frac{1}{\sqrt{\varepsilon_u}}\right) \qquad (6)$$

Assuming that ultimate deflection, δ_{ult}, and steel rupture strain, ε_u, are uncorrelated, Eqs. 4 and 5 may now be applied by replacing Y, X_1, and X_2 by k, δ_{ult}/L_y, and ε_u. Then, given the probability density functions of δ_{ult}/L_y and ε_u, an approximate probability density function of a factor k is obtained.

Figure 6 shows a histogram of steel rupture strain, ε_u for 92 coupon tests listed in Ref. 9. Numbers 6, 7 and 8 bars were used in the tests. Also shown is the corresponding continuous distribution of ε_u based on an assumed normal distribution.

Figure 7 shows the histogram of the ratio δ_{ult}/L_y for the twelve restrained slab tests listed in Table 1, together with the corresponding continuous normal distribution. Figure 8 shows the histogram of the ratio δ_{ult}/L_y and the corresponding continuous distribution for the six simply-supported slab tests listed in Table 2.

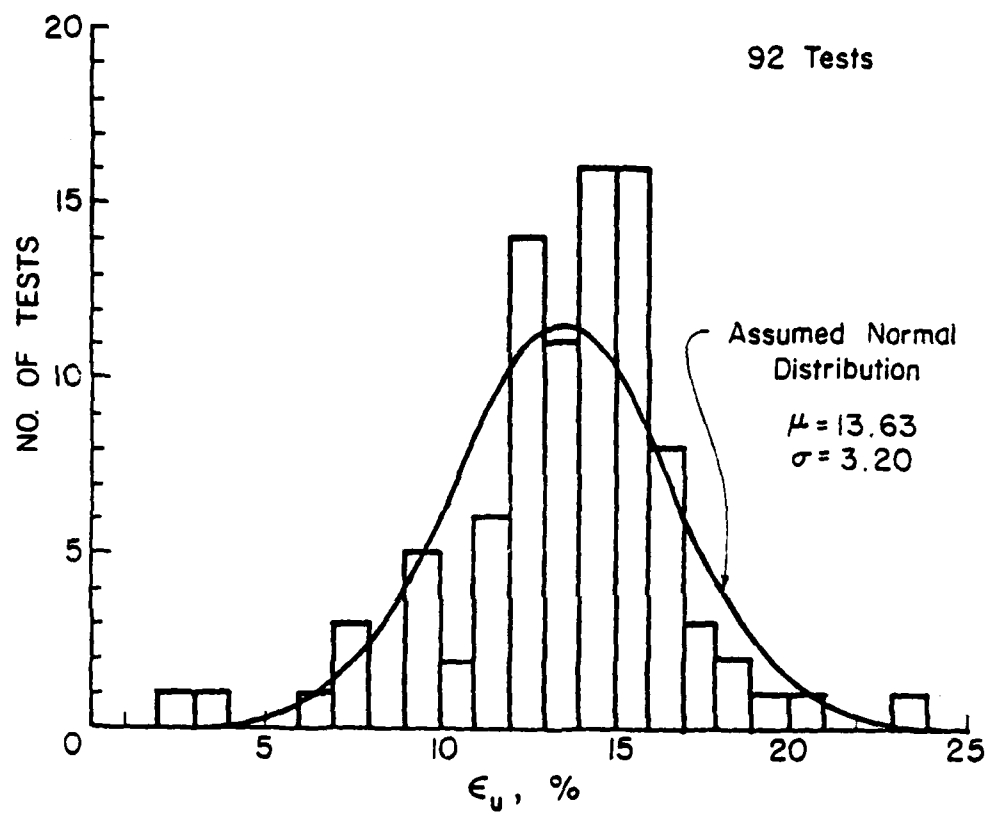Fig. 6 Distribution of Steel Rupture Strain ϵ_u
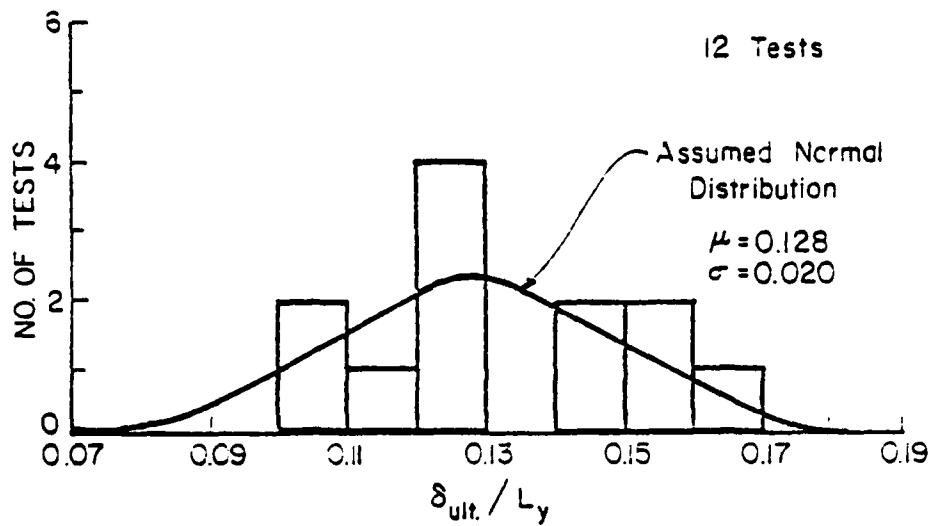from Coupon Test Data (from Ref. 9)
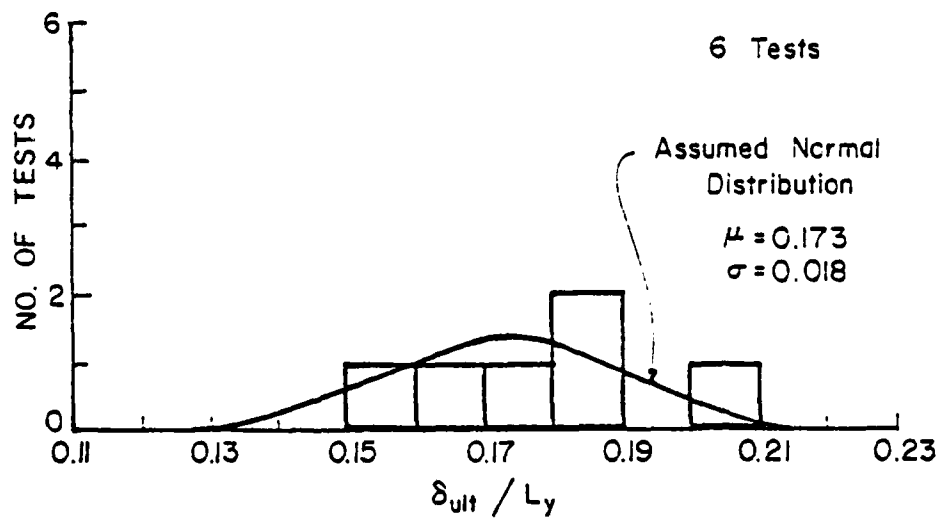
Fig. 7   Distribution of δ_{ult}/L_y for
Restrained Slab Test Data

Fig. 8   Distribution of δ_{ult}/L_y for
Simply-Supported
Slab Test Data

Values of the mean, μ, and standard deviation, σ, for δ_{ult}/L_y, ϵ_u, and k for restrained and simply-supported slabs are summarized in Table 4. The μ and σ for the factor k were calculated from the μ and σ for δ_{ult}/L_y and ϵ_u using Eqs. 4 and 5. Also listed are the corresponding values of k for 90%, 95%, and 99% probabilities of exceedance based on assumed normal distributions. Values of μ, σ, and k are shown for restrained and simply-supported slabs.

It will be noted by comparing Table 3 with Table 4 that, for restrained slabs, smaller values of k leading to more conservative estimates of δ_{ult} are obtained in the first method. In the first method data on ϵ_u are based on only 12 tests. In the second method, data on ϵ_u are obtained from 92 coupon tests. This is apparent from a comparison of values of the standard deviation, σ, of k for both methods. The standard deviation of k for the first method is larger than that for the second method in the case of restrained slabs as shown in Table 3 and Table 4.

This trend is reversed for simply-supported slabs. For this case, the steel rupture strain, ϵ_u, in the first method was assumed to have a constant value of 0.15 for all tests, i.e. no variation, whereas ϵ_u is considered as an independent random variable in the second method. As indicated in Table 3 and Table 4, for simply-supported slabs, smaller values for k are required in the second method for the same probabilities of exceedance.

The first method is based exclusively on measured (or assumed) data from the small number of tests of slabs loaded to incipient collapse. The second method combines the statistics of data on δ_{ult}/L_y from slab tests with the more extensive data on steel rupture strain on , ϵ_u, obtained from coupon tests in addition to the slab tests. To the extent that the second method allows the use of abundant data on ϵ_u to better define the probability density function of ϵ_u, it affords a more accurate method of determining k values associated with specific probabilities of exceedance.

2.4 Proposed Design Criteria for Incipient Collapse Deflection

It is recommended that values of k correponding to a 99% probability of exceedance as listed in Table 4 be selected for the purpose of calculating incipient collapse deflection. Ultimate deflection, i.e., incipient collapse deflection of a slab is calculated using Eq. 1 with the value of k selected depending on support conditions of the slab.

It should be noted that the effect of aspect ratio is not included in Eq. 1. Evaluation of the effect of this parameter would require, in the absence of an acceptable analytical tool, the testing of a series of slabs with only the aspect ratio

Table 4 Values of k for Selected Probabilities of Exceedance - Steel Rupture
Strain Considered as Independent Random Variable

Support Conditions	ε_u		δ_{ult}/L_y		k		Values of k Corresponding to Probabilities of Exceedance*		
	μ	σ	μ	σ	μ	σ	90 %	95 %	99%
Restrained 12 Tests	0.136	0.032	0.128	0.020	0.354	0.068	0.267	0.242	0.196
Simply Supported 6 Tests			0.173	0.018	0.479	0.074	0.384	0.358	0.307

*Probability that the calculated value of $\delta_{ult} = k \, L_y \, \sqrt{\varepsilon_u}$ will be exceeded.

-20-

being varied. The lack of this type of experimental data precluded the investigation of this effect.

The values of k determined here are based on two-way slab tests. As stated in the Phase I report, very little information is available on one-way restrained strips tested to incipient collapse under uniformly distributed load. A significant number of tests were carried out using two equal loads at the middle-third points. However, none of these tests was carried to incipient collapse. In the absence of specific test data, it is believed that the use of k values derived for two-way slabs for the case of one-way slabs is reasonable. It is pointed out that the single-cable model used in deriving Eq. 1 represents a closer approximation of the one-way slab strip.

Equation 1 cannot be applied to one-way simply-supported, i.e., unrestrained, slabs since tensile membrane action cannot develop in this type of slab. The ultimate deflection capacity of simply-supported one-way slabs is determined by the flexural rotation capacity of the hinging region or yield line. In uniformly loaded slabs this yield line occurs at midspan. The rotational capacity of interest is that associated with the maximum capacity of the slab, beyond which a rapid decrease in load occurs.

Derivation of an expression for the maximum deflection and corresponding edge rotation of uniformly loaded, simply-supported one-way slabs is presented in Appendix B. The proposed expression, which is based on a number of simplifying assumptions, gives the ultimate edge rotation as a function of several variables. These variables are maximum usable strain in concrete, slab span, distance between top and bottom slab reinforcement, yield stress of reinforcement, and post-yield slope of stress-strain curve of reinforcement. A limited number of combinations of these quantities are examined in Appendix B to illustrate the use of the proposed expression. For the cases examined, edge rotations in the range of 1 to 2 degrees were obtained.

Similarly, Eq. 1 cannot be applied to slabs restrained along two adjacent sides only, since no tensile membrane can form in either direction. In the case of a rectangular slab restrained on three sides only, the slab should be treated as a one-way slab restrained along two opposite sides, since tensile membrane action can form in only one direction.

Based on the preceding observations, values of the factor k recommended for use with Eq. 1 are listed in Table 5. Only the most commonly encountered support conditions for rectangular slabs are shown. Where Eq. 1 is not applicable because no tensile membrane action can be developed in a slab, the deflection corresponding to the formation of a yield-line mechanism represents the ultimate deflection of the slab.

-21-

Table 5 Recommended k Values for Use in Eq. 1*
Corresponding to Selected
Slab Support Conditions

Support Conditions	Values of k	Support Rotation Angle, in Degrees	
		For Arbitrary ϵ_u	For ϵ_u = 0.14
Two-Way Slabs			
a) Restrained	0.20	$\tan^{-1} 0.40 \sqrt{\epsilon_u}$	8.5
b) Simply-Supported (Unrestrained)	0.30	$\tan^{-1} 0.60 \sqrt{\epsilon_u}$	12.7
One-Way Slabs			
a) Restrained	0.20	$\tan^{-1} 0.40 \sqrt{\epsilon_u}$	8.5
b) Simply-Supported (Unrestrained)	Eq. 1 Not Applicable	Eq. 1 Not Applicable	–
Two Way Slabs with Three Sides Restrained	Treated as One-Way Restrained Slabs, k = 0.20	$\tan^{-1} 0.40 \sqrt{\epsilon_u}$	8.5
Two-Way Slabs with Three Sides Restrained	Eq. 1 Not Applicable	Eq. 1 Not Applicable	–

*$\delta_{ult} = k \, L_y \sqrt{\epsilon_u}$

3. DESIGN CONSIDERATIONS

3.1 Introduction

To develop effective tensile membrane action, design require-
ments beyond those associated with development of a flexural
yield-line mechanism in a slab should be considered. It is
implicitly assumed that adequate provision is made so that
premature failure in shear or bond does not occur before the
development of the yield-line mechanism.

In tensile membrane action, a slab acts essentially as a cable,
with cracks penetrating the entire slab thickness. A designer
must carefully detail reinforcement to assure development of its
full tensile strength. This chapter examines the necessary re-
quirements to achieve this goal.

3.2 Importance of Details in Structures

Details in structures have always commanded the attention of en-
gineers concerned with unusual loads, whether in terms of magni-
tude or character or both. It has been pointed out that the
difference between a "good" structure and a "bad" structure
lies mainly in details and in structural concept (10). Proper
detailing allows a structure to survive loadings not normally
accounted for in design or expected only infrequently. The
major objective in detailing for satisfactory performance under
unusual loading is to provide adequate ductility and energy dis-
sipation capacity.

Detailing requirements covering structures subjected to normal
live loads as well as dynamic loads are contained in most model
building codes (11-15). The basic detailing requirements
designed to provide ductility may be stated as follows:

1. All required minimum reinforcement should be
    continuous through joints.

2. All principal reinforcement in critical regions,
    particularly at joints and tension splices,
    should be enclosed by confinement reinforcement.

The first of these provisions stipulates continuity throughout
the structure and allows for redistribution of loads should
this become necessary. The second requirement provides for the
formation of flexural "hinges" and plastic deformation. This
particular requirement would normally apply to beams and beam-
column frames. Except where the special effort is justifiable,
the use of transverse or confinement reinforcement (e.g.,
lacing, stirrups, etc.) in slabs is not common. Requirements
contained in Appendix A of the ACI Building Code (11), intended
for earthquake-resistant structures, embody both of the above
detailing considerations. Essential requirements affecting
slabs and beams are summarized in Figs. 9 and 10.

-23-
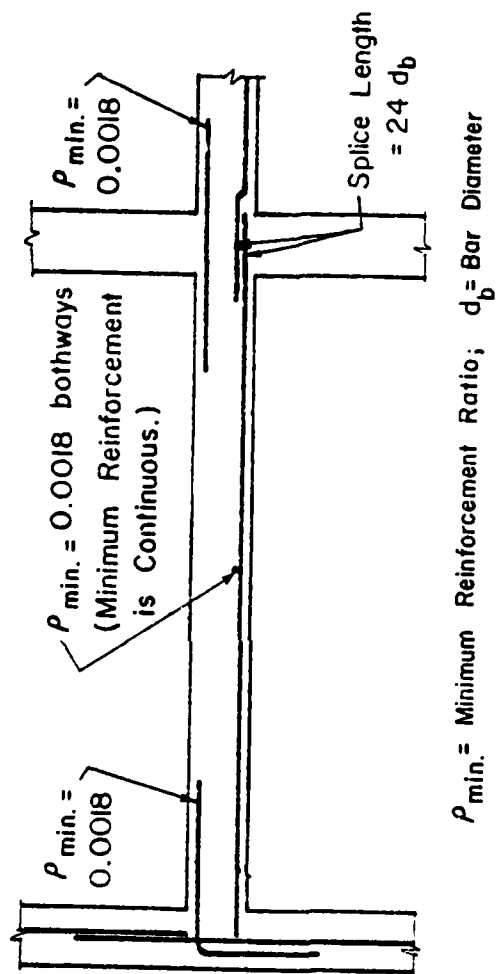
Fig. 9 Detailing Requirements for Slabs
Appendix A, ACI 318-77

$\rho_{min.}$= Minimum Reinforcement Ratio; d_b= Bar Diameter

$\rho_{min.}$=
0.0018

$\rho_{min.}$ = 0.0018 bothways
(Minimum Reinforcement
is Continuous.)

Splice Length
= 24 d_b

$\rho_{min.}$=
0.0018

-24-
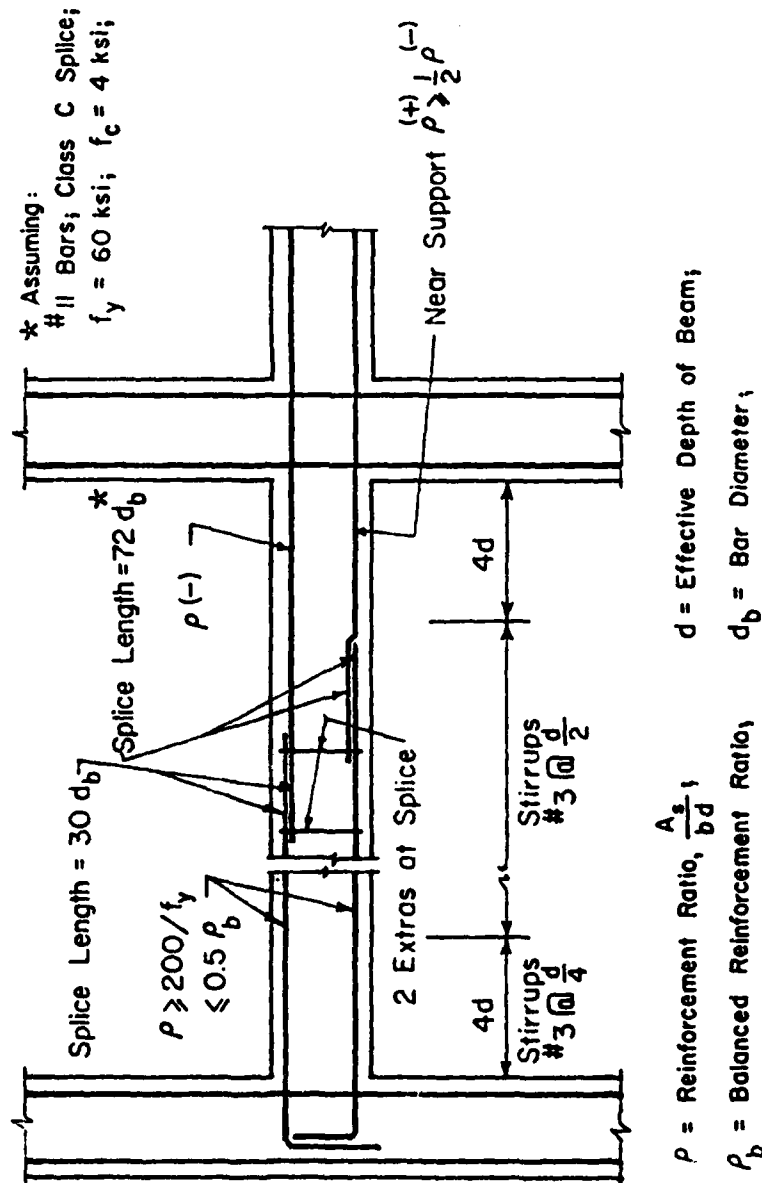
Fig. 10 Detailing Requirements for Beams
Appendix A, ACI 318-77

ρ = Reinforcement Ratio, $\dfrac{A_s}{bd}$;   d = Effective Depth of Beam;

ρ_b = Balanced Reinforcement Ratio, d_b = Bar Diameter;

*Assuming:
#11 Bars; Class C Splice;
f_y = 60 ksi; f_c = 4 ksi;

Splice Length = 30 d_b

Splice Length = 72 d_b*

ρ(-)

$\rho \geqslant 200/f_y$
$\leqslant 0.5 \rho_b$

2 Extras at Splice

Stirrups
#3 @ $\dfrac{d}{4}$

Stirrups
#3 @ $\dfrac{d}{2}$

4d

4d

Near Support $\overset{(+)}{\rho} \geqslant \dfrac{1}{2} \overset{(-)}{\rho}$

-25-

In detailing for tensile membrane action, the basic requirements listed above must be supplemented to provide for special features associated with the tensile membrane mechanism of resistance.

In the tensile membrane range, ductile performance in slabs is measured not so much in terms of the flexural rotational capacity of hinging regions near the supports as by the deflection capacity of the slab as a whole. As such, the performance of the slab becomes less dependent on such considerations as the shear strength of the critical flexural regions of the slab and is influenced more by the ductility of the reinforcement itself in tension as well as the integrity of its anchorage.

3.3 Development of Tensile Membrane Action

The basic requirements for development of tensile membrane action in slabs are twofold. First, the slab must possess tensile strength and extensibility beyond the snap-through range, or Point E in Fig. 1. Second, either the slab itself or the surrounding structure must be able to provide the necessary horizontal restraint. Extensibility of the reinforcement depends on its breaking strain. This value may be determined from simple tests of the reinforcing bars used. Alternatively, a reasonably conservative value may be assumed.

Concerning the fundamental question as to whether tensile membrane action should be considered, Wood (16) remarked that no allowance for membrane action should be made if the supporting beams are involved in the mechanism of collapse and hence provide no restraint for development of tensile membrane action.

A vital requirement in designing for tensile membrane action in a slab is that the reinforcement be carefully detailed so that it is effectively continuous. Anchorage lengths for bars within supports must be capable of developing the full tensile capacity of the reinforcement. Thus, anchorage of the tensile reinforcement in regions or members providing adequate horizontal and vertical restraint is the primary requirement for the development of incipient collapse condition as defined here. If anchorage is inadequate to develop the full tensile capacity of a bar, incomplete tensile membrane action will result.

Stages of premature failure due to inadequate anchorage are shown in Fig. 11. Failure occurs by tearing out of the bottom bars from the slabs, starting at the cut-off points for negative reinforcement and extending back to supports, with collapse occurring when the final short lengths are pulled out at the supports.

For a simply-supported two-way slab without lateral restraint, no support element provides anchorage space at edges of the

Compressive Membrane Phase

Tensile Membrane Action Established
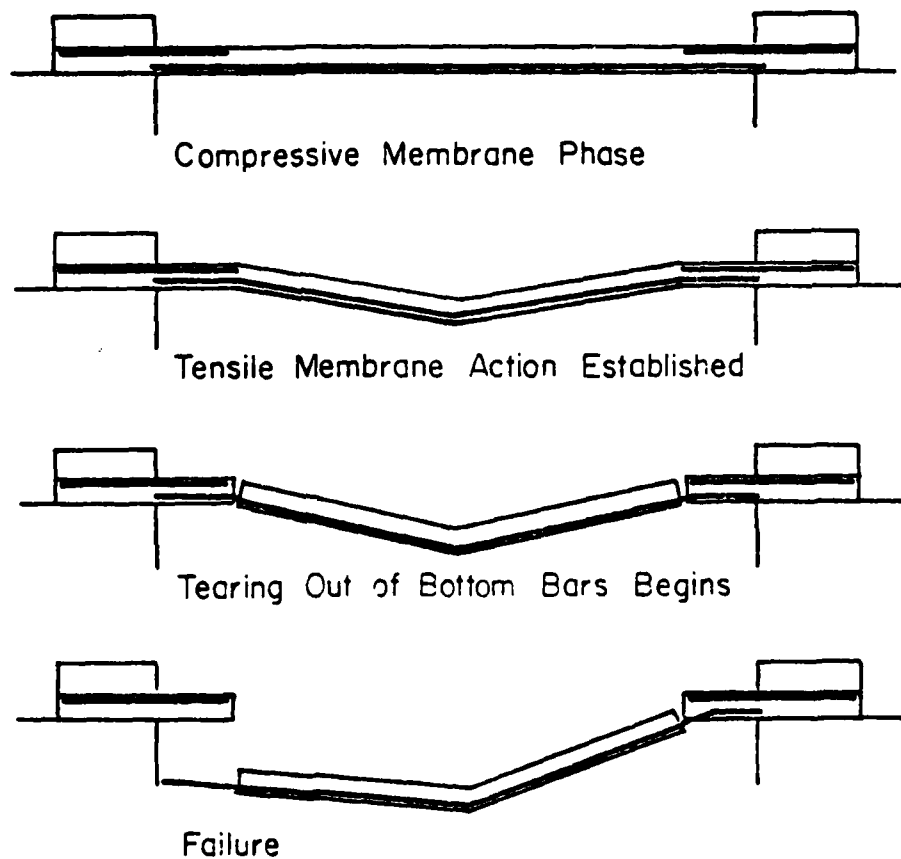
Tearing Out of Bottom Bars Begins

Failure

Fig. 11   Premature Tensile Membrane Collapse Due to
Bond or Anchorage Failure

slab. In this case, the circumferential compression ring that forms near the periphery of the slab provides the anchorage space. At the edges of slab, the reinforcing bars should be securely hooked around longitudinal bars spanning in the other direction.

Once the basic requirements for tensile membrane action are met, the problem of achieving economy by minimizing the amount of flexural reinforcement may be considered. Minimum weight design requires uniform dissipation of energy per unit volume of reinforcement everywhere. This implies the provision of steel reinforcement in proportion to the magnitude of the strains in the slab.

One of the best and simplest reinforcement arrangements is to have bars parallel to the edges but concentrated in the middle strip. Tests (17) indicate that this arrangement strengthens the tensile membrane action at the center and increases the circumferential compression in the outer regions. The reduction of reinforcement in the outer regions is not detrimental since the yield moment is raised considerably because of compression in the region.

The results of a few tests by Taylor, Maher, and Hayes (17) indicate that stopped-off and bent-up bars tend to reduce the tensile membrane capacity of slabs. Until more information is available on the effects of these design features on slab performance in the tensile membrane action range, it is recommended that they be avoided.

Because cracks penetrate the entire slab thickness in the tensile membrane range, little reliance can be placed on lap splices located in this region. In view of this, lap splices should be avoided in reinforcement intended to contribute to tensile membrane action. Near the periphery of the slab, where some circumferential compression may still be present in the tensile membrane range, the reinforcement may perhaps be spliced, provided the splices are staggered.

3.4 Membrane Action to Resist Blast Loads

The design criteria developed for incipient collapse deflection of conventionally reinforced concrete slabs are based on data obtained from statically loaded small-scale slab specimens. Investigators whose works have been reviewed did not use any special construction details to obtain relatively large deflections at incipient collapse. No splicing of reinforcement was used since the slab specimens were of relatively small size. Most specimens were singly reinforced and had no confinement reinforcement.

Where a slab is specifically designed to resist blast loading, additional design requirements may have to be considered. Thus,

the need to prevent large gaps in the slab due to loss of concrete fragments under blast loading will require effective confinement of concrete. The degree of confinement required will vary with the intensity of the blast loading.

Where debris hazard from direct spalling, i.e., due to the tensile forces normal to the plane of the slab resulting from the reflection of the blast pulse from the far surface of the slab, is to be minimized, some means of retaining or containing potential loose fragments of concrete will have to be provided.

The principal effects of an explosion are blast pressure and primary fragments. Both effects are dynamic in nature and cause a resisting element to deflect (with well-defined cracks) until such time that: (1) the strain energy of the element is developed sufficiently to balance the kinetic energy produced by the applied load and the element comes to rest, or (2) fragmentation of the concrete occurs resulting in either partial or total collapse of the element. The incipient collapse deflection attainable is a function of the span, steel breaking strain, and details of the reinforcement used in a particular design.

NAVFAC P-397 (1) stipulates that ductility of flexural reinforcement and integrity of concrete between two layers of flexural reinforcement are vital for successful structural performance of a slab element under blast pressure. Laced reinforcement is required for structures located inside the immediate high blast intensity area (high-pressure design range). Structures located in the intermediate- and low-pressure design ranges can be designed without lacing since deflection required to absorb the loading is considerably smaller than the deflection that can be realized with laced concrete elements.

3.5 Summary of Design Considerations

Principal design considerations associated with development of tensile membrane action in conventionally reinforced concrete slabs are summarized below.

1. Principal flexural reinforcement should be continuous throughout the spans of slabs. No cut-off or splicing of the reinforcement should be allowed within the span. In the tensile membrane action range, cracks in the central region of a slab penetrate the entire thickness and transfer of stress between reinforcing bars and concrete may be destroyed. In the outer, peripheral, region of a slab where circumferential compression occurs, splices may be used provided these are staggered. The effect of such splices on slab tensile membrane behavior, however, needs to be investigated.

The use of double (i.e., top and bottom) reinforcement is desirable from the point of view of confinement of the concrete between the two layers.

2. Adequate anchorage of the main flexural rein- forcement in boundary support elements must be provided. Such anchorage should be sufficient to develop the strength of the reinforcement in tension.

   For an unrestrained, simply-supported, two-way slab, the compression ring that forms near the periphery of the slab during tensile membrane action provides space for anchorage, since no boundary support element is available. At edges of the slab, bars should be hooked around bars spanning in the other direction. Also, adequate concrete cover should be provided in compression ring regions. Diagonally arranged reinforcement may be a possible means of pro- viding improved bar anchorage at the corners of the slab where the compression ring forms. In this case, the slab should be square or nearly square in plan.

3. An adequate amount of reinforcement relative to the gross concrete section should be provided to ensure desired tensile membrane strength. This maximum strength, corresponding to Point F in Fig. 1, would normally be comparable to the com- pressive membrane strength, Point D in Fig. 1 (4).

4. The slab support system, the surrounding beams in the general case, should remain intact when the collapse mechanism forms in the slab.

Particular emphasis should be placed on Item 2. No matter how well a slab is reinforced in its clear span, collapse can occur if adequate anchorage of the reinforcement is not provided. Therefore, effectiveness of end anchorage of the reinforcement is vital to develop the tensile membrane load capacity and incipient collapse deflection of slabs.

Detailing requirements for proper anchorage of slab reinforce- ment are explored in detail in the next chapter.

4. END ANCHORAGE REQUIREMENTS FOR TENSILE
MEMBRANE ACTION IN SLABS

In the following sections, several reported slab tests are re-
viewed, with emphasis on the details of reinforcement anchorage
used in the specimens. Next, the results of recent tests on
development lengths of bars, with particular reference to rele-
vant provisions in the ACI Code (11), are discussed. Finally,
recommendations for anchorage details to develop tensile mem-
brane action and incipient collapse deflection in slabs are
proposed.

4.1 Review of Selected Slab Test Data

Only those tests in which specimens were loaded to incipient
collapse are reviewed. Types of anchorage systems used in
these tests are evaluated in terms of their applicability to
full-scale slabs. Overall aspects of the tests are discussed
in Phase I (2).

Black's Work. The tests reported by Black (5) were performed on
small-scale slabs approximately 1/8 the size of the prototype.
At this scale, size effects may be significant, especially with
respect to anchorage capacity.

The embedment length relative to bar size used in the tests was
excessive compared to current code requirements. An embedment
length of about 8 in. or 100d relative to the 0.08-in. diameter
steel wire, in addition to a large bent portion, was used in
the tests. The anchorage detail used is shown in Fig. 12.
Failure of the specimens resulted from tensile rupture of the
individual wires around the periphery of the slab.

Keenan's Work. The specimens tested by Keenan (4) had continu-
ous shear reinforcement (stirrups) near the supports. These
were placed in a zigzag fashion around the top and bottom longi-
tudinal bars as shown in Fig. 13. This type of reinforcement
would not be considered conventional. All edges were fully
clamped and laterally restrained against outward movement with
a pair of bolts and steel channels. Over the supports, the bars
were securely hooked around longitudinal bars spanning in the
other direction.

The slabs were loaded well into the tensile membrane range.
Failure generally occurred by one or more reinforcing bars rup-
turing in tension near the center of a support edge.

Park's Work. The support system for slabs used in Park's tests
(18) was similar to that used by Keenan. Slab edges which were
to be fully fixed against rotation and translation were clamped
to the supporting frame by two hold-down bolts.
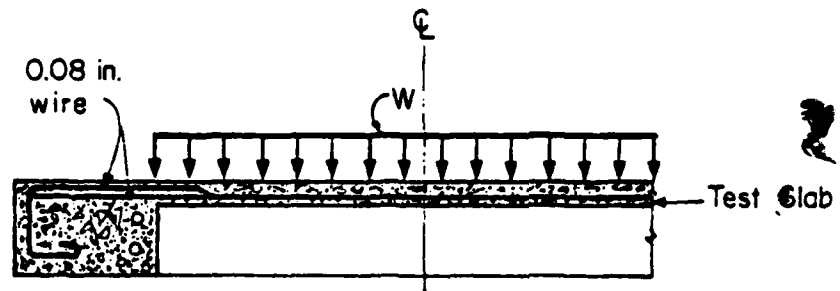
Fig. 12 Reinforcement Anchorage
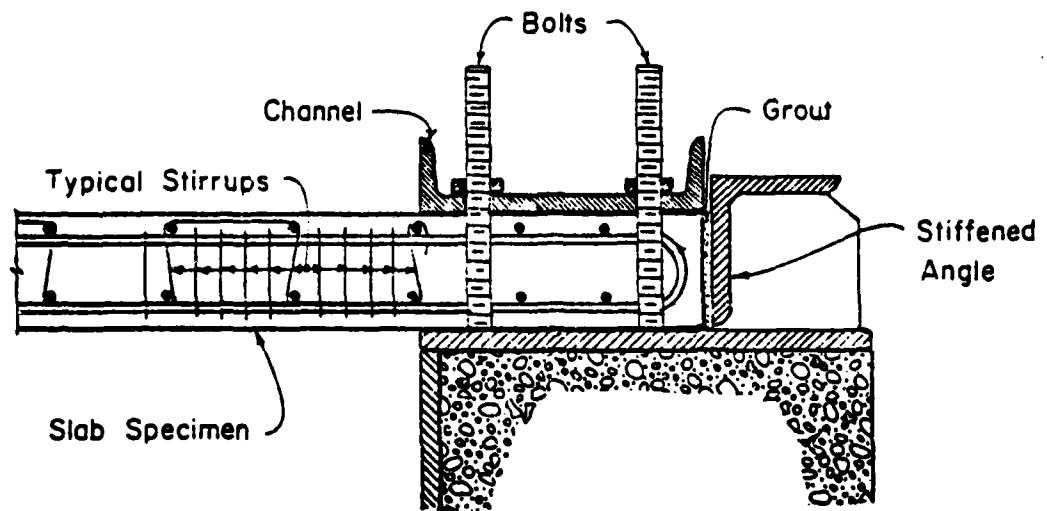Detail - Black
(from Ref. 5)

Fig. 13 Reinforcement Anchorage Detail - Keenan
(from Ref. 4)

Hopkins and Park's Work. The test by Hopkins and Park (19) was conducted on a 1/4-scale, nine-panel reinforced concrete slab, shown in Fig. 14. Top bars were cut-off at about a third of the single panel span from the supports. The bottom bars were continuous over the entire three-panel span. Top and bottom bars of all panels were approximately 0.16% and 0.15% of the gross concrete area, respectively. No information on the manner in which the slab bars were anchored in the edge beams is given in the paper. However, the main interest in the test is in the behavior of the center panel. Under a uniform load over the entire specimen, the critically loaded center panel exhibited clear evidence of tensile membrane action before failure in spite of the relatively small amount of reinforcement used. The edge panels did not develop tensile membrane action. Tensile membrane action in the center panel was attributed mainly to the presence of surrounding panels and beams which were fully mobilized to form a compression ring. Furthermore, these surrounding panels allowed ample anchorage space for the reinforcement of the center panel. These tests did not, however, provide information on the behavior of the edge panels in the tensile membrane range.

Brotchie and Holley's Work. The slab specimens tested by Brotchie and Holley (6) were 15 in. square in plan, with a thickness of 0.75 in. and reinforced with No. 13 wire (0.0915-in. diameter). The slabs were clamped along the edges against elongation and rotation by a top plate, a bottom plate, and by epoxy resin, as shown in Fig. 15.

The slab extended approximately 7 in. into the support all around. The clamped portion of the slab provided an anchorage length of about 75d. Reinforcement consisted of a single bottom layer of steel wires distributed uniformly and equally in each direction. Amount of reinforcement each way was either 1% or 2% of the gross concrete section.

All of the above tests in which the slabs were loaded to incipient collapse used either too long an anchorage length or restraint conditions along the edges. These conditions are not normally found in practice. In view of this, little information can be derived from these tests with respect to adequacy of normal anchorage details. The adequacy of code-prescribed anchorage details for the specific purpose of developing significant tensile membrane capacity in slabs, therefore, requires experimental verification.

4.2 Development Length for Deformed Bars in Tension

Results of a large number of tests dealing with development length, splices and hooks for reinforcing bars in tension have been reported in the literature. Development length is that length of reinforcing bar necessary to transfer the force corresponding to a specified stress level from the bar to the
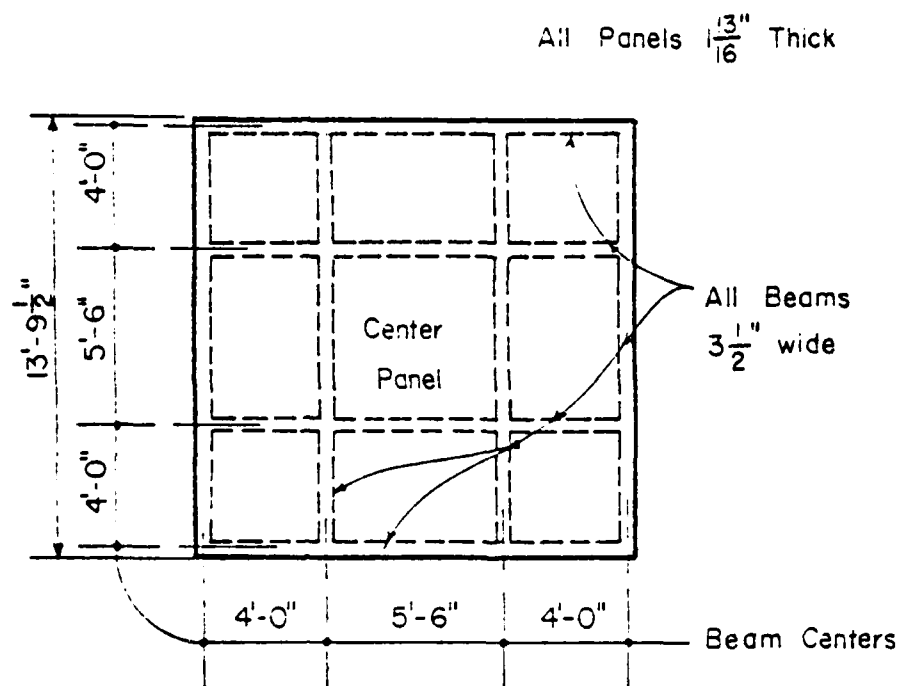
All Panels $1\frac{13}{16}$" Thick

Center Panel

$13'-9\frac{1}{2}$"

4'-0"
5'-6"
4'-0"

All Beams $3\frac{1}{2}$" wide

4'-0"   5'-6"   4'-0"

Beam Centers

Fig. 14 1/4-Scale Beam-Slab System
Tested by Hopkins and Park
(from Ref. 19)

Clamping
Force

Epoxy Resin Fill

Top Plate

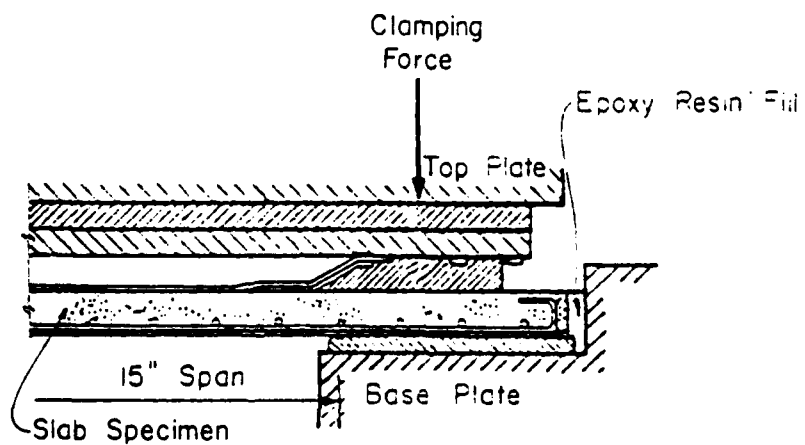15" Span

Base Plate

Slab Specimen

Fig. 15 Slab Edge Detail - Brotchie
and Holley (from Ref. 6)

concrete. Parameters that may affect anchorage are: concrete strength, yield stress of reinforcing bars, bar diameter, position of bars, embedment length, concrete cover, bar spacing, transverse reinforcement, confinement, hook geometry, and aggregate properties, i.e., lightweight vs. normal weight concrete. Confinement may take the form of a compression stress field, stirrups, hoops, ties or spirals.

4.2.1 **ACI 318-77 Recommendations.** The most commonly referred to design provisions governing development lengths, splices and hooks in reinforcing bars are those found in the American Concrete Institute's "Building Code Recommendations for Reinforced Concrete". The latest edition of this standard was issued in 1977, i.e., ACI 318-77 (11). The current ACI provisions on development length are based mainly on the work of Ferguson and associates at the University of Texas in Austin (20, 21).

Most tests on development length of reinforcing bars have been performed on beams. Figure 16 shows a specimen used by Ferguson and Thompson (20, 21). The specimen was a new type test beam designed to place the development length of the bar in a negative moment region. The simple-span beam, which had been used in earlier tests, was extended beyond a support in one direction to provide a cantilever overhang. This permitted the tested bar to be located in a negative moment region. Since the test bar was removed from neighboring bars, reaction, and load points, this test setup represented the simplest bond condition along the development length L.

The tests by Ferguson and Thompson considered the following variables: (1) concrete strength, (2) clear cover over bars, (3) development length and bar size, (4) end anchorage or hooks, (5) multiple cut-offs, (6) stirrups, (7) beam width, (8) depth of concrete placed below bar, (9) relationship of crack width to that in pure flexure. Geometrical and material properties of the test specimens are summarized in Table 6.

Among the significant findings based on the tests are:

1. The required development length increases significantly with the diameter of the bar; e.g. 20 diameters for a No. 3 bar and 54 diameters for a No. 11 bar.

2. The bond strength increases with increased concrete strength, approximately in proportion to the square root of f'_c.

3. Width of beam is a significant factor affecting bond strength developed, narrower beams failing at bond stresses 7 to 20 percent lower.
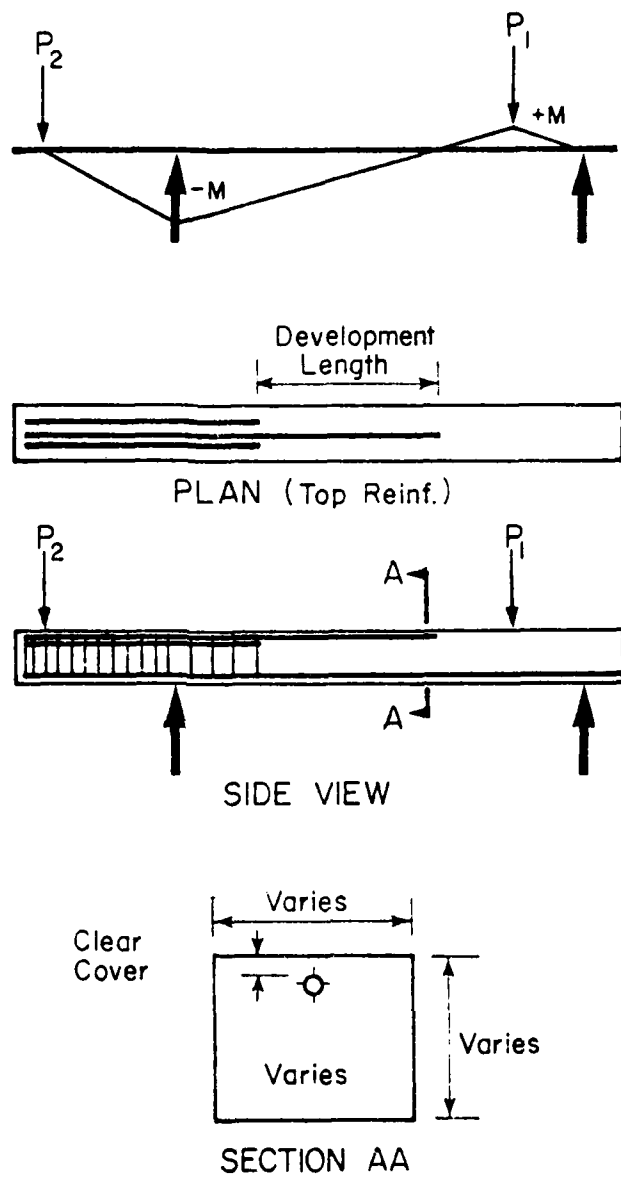
P₂   P₁

+M

−M

Development
Length

PLAN (Top Reinf.)

P₂   P₁

A

A

SIDE VIEW

Varies

Clear
Cover

Varies

Varies

SECTION AA

Fig. 16   Typical Development Length Test
Specimen - Ferguson and Thompson
(from Ref. 20)

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Table 6 Geometrical and Material Properties of Test Specimens on Development Length

Investigators	Bar Size d_b in.	No. of Specimens n	Concrete Compression Strength f'_c psi	Steel Yield Strength f_y ksi	Beam Width b in.	Beam Depth D in.	Concrete Cover c_c/d_b	Effective Depth d/d_b	Bar Spacing c_s/d_b	Embedment Length L/d_b
Chamberlin (22)	#4 0.500	88	3680 ~5870	50.0	6.0	6.0	2.0 ~ 4.5	7.0 ~ 9.5	1.0 ~ 5.0	6.0 ~21.3
	#6 0.750	12	4470	50.0	9.0	9.0	1.3	10.2	2.0 ~ 5.0	21.3
Mathey and Watstein (23)	#4 0.500	9	3675 ~4625	114.7	8.0	18.0	3.5	32.0	16.0	14.0 ~ 34.0
	#8 1.000	9	3495 ~4485	97.0	8.0	18.0	1.5	16.0	8.0	7.0 ~ 34.0
Ferguson and Thompson (20)	#3 0.375	8	2380 ~3850	82.0	6.0 ~10.0	4.3 ~ 5.2	1.8 ~ 4.5	6.7 ~11.5	16.0 ~26.7	20.0 ~ 40.0
	#7 0.875	60	2380 ~5670	87.5	12.0 ~18.0	6.0 ~11.2	0.9 ~ 3.1	3.4 ~10.2	13.7 ~20.6	18.0 ~ 48.0
	#11 1.410	8	3020 ~3620	86.0	18.0	9.7 ~11.0	1.0 ~ 1.9	4.5 ~ 6.4	12.8	32.0 ~ 48.0
Ferguson and Thompson (21)	#11 1.410	32	3000	86.0	18.0 ~24.0	13.0 ~18.0	1.1 ~ 3.5	8.2 ~10.9	3.8 ~17.0	24.0 ~ 48.0
Typical Slab (24)	#3 0.375 #9 ~ 1.128					4.0 ~10.0	0.6 ~ 2.0	6.0 ~16.0	9.0 ~36.0	

*Thickness of concrete cover measured from extreme tension fiber to outermost surface of bar divided by bar diameter
**Distance from extreme compression fiber to centroid of tensile bar divided by bar diameter
***Center to center bar spacing or twice the distance measured from lateral concrete surface to centroid of outermost bar divided by bar diameter

Included in Table 6 are data from earlier tests by Chamberlin
(22) and Mathey and Watstein (23). These investigators used
simple span beam specimens in which the development length was
located in the positive moment region near the supports.

Normally, the reinforcement size used in slabs ranges from No. 3
to No. 9, slab thickness from 4 to 10 in., concrete cover from
0.6 to 2.0 times bar diameter, effective depth from 6 to 16
times bar diameter, and bar spacing from 9 to 36 times diameter
(24). When these ranges are compared with the values used in
development length tests, as indicated in Table 6, it will be
noted that the ranges of specimen parameters shown in Table 6
are representative of practical slab sizes, except concrete
cover. As indicated in Table 6, the concrete cover-to-bar
diameter ratio used in reported tests lie in the upper range of
values of practical slabs.

It is of interest to determine if expressions based on the
series of tests listed in Table 6 can be properly applied to
cases in which the concrete cover is less than those used in
the tests and particularly to slabs loaded to incipient col-
lapse. The need to develop more information on anchorage of
bars in slab-type specimens with low concrete cover ratios is
one reason for the recommendation to undertake an experimental
investigation. This is discussed briefly in the Appendix.

Results of more recent tests by Jirsa and associates and others
(25, 26, 27, 28, 29) are reported in an ACI Committee 408
report (30). These are discussed in the following section.

In ACI 318-77, the minimum basic development length required
for No. 11 bars and smaller is expressed as a function of bar
area, bar diameter, concrete strength, and yield stress of
reinforcing bars, as follows:

$$\ell_d = 0.04\ A_b f_y\ \sqrt{f_c'} \geq 0.0004\ d_b f_y \quad (in.), \qquad (7)$$

where: A_b = cross-sectional area of bar, in.^{2}

   d_b = bar diameter, in.

   f_c' = concrete compressive strength, psi

   f_y = yield stress of reinforcing bar, psi

Effects of position of reinforcing bar in the section, concrete
aggregate properties and confinement are considered by means of
specified factors to be applied to the basic development length.
The conditions under which these factors apply and the corres-
ponding values are specified as follows:

(1) Top reinforcement which is
horizontally placed so that
more than 12 in. of concrete
is cast in the member below
the reinforcement,   1.4

(2) Reinforcement with f_y
greater than 60,000 psi, $(2-60,000/f_y)$

(3) Lightweight aggregate
concrete when its average
splitting tensile strength
f_{ct} is specified, $6.7 \sqrt{f_c'}/f_{ct}$

When f_{ct} is not specified

   a. all-lightweight concrete,   1.33

   b. sand-lightweight concrete,   1.18

(4) Reinforcement being developed
in length under consideration
and spaced laterally at least
6 in. on center with at least
3 in. clear from face of mem-
ber to edge bar, measured in
direction spacing,   0.80

(5) Reinforcement in a flexural
member in excess of that re- $\dfrac{(A_s \text{ required})}{(A_s \text{ provided})}$
quired by analysis,

(6) Reinforcement enclosed within
spiral reinforcement not less
than 1/4 in. diameter and not
more than 4 in. pitch,   0.75

The expression for basic development length, such as given by
Eq. 7, assumes a uniform distribution of bond stress along the
development length and a maximum tensile stress to be developed
equal to 1.25 the yield stress of the bar. Thus, using the ex-
pression for ultimate bond stress specified for bottom bars in
ACI 318-63 (an earlier version of Ref. 11), i.e.,

$$u = 9.5 \sqrt{f_c'}/d_b \text{ (psi)}, \tag{8}$$

one obtains

$$\pi d_b (9.5 \sqrt{f_c'}/d_b)\, \ell_d = A_b (1.25 f_y). \tag{9}$$

from which

$$\ell_d \cong 0.04\, A_b f_y / \sqrt{f_c'} \text{ (in.)} \tag{7}$$

It is important to note that factors such as bar spacing, concrete cover and transverse reinforcement are not considered in the above expression for basic development length. Because slabs are relatively thin and generally do not have web reinforcement, the degree of confinement of main flexural steel bars is not as good as in deep members with web reinforcement. The need to ensure proper anchorage of slab bars is one of the reasons why beams of adequate section should be provided along discontinuous edges of slabs.

4.2.2 ACI Committee 408 Recommendations. Based on a re-evaluation of earlier tests as well as an evaluation of recent tests, recommendations on development length, splices, and standard hooks for deformed bars in tension were recently published by ACI Committee 408 – Bond and Development of Reinforcement (30). In the recommendations, basic development length ℓ_{db} for Grade 60 deformed bars in tension is given as:

$$\ell_{db} = \frac{5500\, A_b}{\phi K \sqrt{f_c'}} \text{ (in.)}, \tag{10}$$

where: K = the smaller of (a) $C_c + K_{tr}$ or (b) $C_s + K_{tr}$

C_c = thickness of concrete cover measured from extreme tension fiber to center of bar, in.

C_s = the smaller of the cover to the center of bar measured along the line through the layer of bars, or half the center-to-center distance of bars in the layer as illustrated in Fig. 17, in.

K_{tr} = an index of the transverse reinforcement provided along the anchored bar, $A_{tr} f_{yt}/1500s$, in.

ϕ = strength reduction factor for development length and splices = 0.8

A_{tr} = area of transverse reinforcement crossing plane of splitting (a) parallel to the layer of bars for C_c or (b) total area of transverse reinforcement divided by n for C_s, in.^{2}

$$n \quad = \text{number of bars in layer}$$

$$f_{yt} \quad = \text{specified yield stress of transverse reinforcement, psi}$$

$$s \quad = \text{maximum spacing of transverse reinforcement within } \ell_d, \text{ center-to-center, in.}$$

The required development length, ℓ_d, for any particular application is computed as the product of the basic development length and applicable modification factors. Modification factors to account for effects of yield stress of reinforcement other than Grade 60, top horizontal reinforcement and light-weight aggregate are given. These effects are essentially the same as those appearing in ACI 318-77, except that the values of the factors are different. The applicable modification factors are:

(1) Reinforcement having yield
stress other than 60,000 psi, $f_y/60,000$

(2) Top horizontal reinforcement
where more than 12 in. of fresh
concrete is cast in the member
below the bar,   1.30

(3) Lightweight aggregate to re-
place all or a portion of the
aggregate,   1.25

(4) Reinforcement in a flexural
member in excess of that A_{sr} (required)
required,   ────────────────
                                            A_{sp} (provided)

    4.2.3 Basis and Development of ACI Committee 408 Recommen-
dations. In a discussion of these recommendations, Jirsa, Lutz
and Gergely (31) compared the recommended provisions with
current ACI Building Code requirements (ACI 318-77). A major
improvement over the ACI 318-77 expression for development
length found in the ACI Committee 408 report is the inclusion
of the effects of new parameters that recent tests and a re-
evaluation of previous tests have shown to be significant.
Thus, the ACI Committee 408 equations take into account the
effects of concrete cover, bar spacing and transverse rein-
forcement on the anchorage strength of bars in tension. These
parameters do not appear in the current ACI Code equations.

A comparison of the average bond stress as predicted by ACI
318-77 with test data on development length was made by Jirsa,
et al (31). For this purpose, data on 254 development length
tests, as summarized by Orangun, Jirsa and Breen (25), was
used. Figure 18 shows a histogram for the ratio u_{test}/u_{cal}.

As can be seen, the variability is fairly large and a significant number of tests gave average bond stresses less than calculated. Examination of the test data indicates that the average bond stress recorded in the tests tends to be lower than predicted values where concrete cover over the bars or spacing between bars is small. For large concrete covers and spacings, and for cases where transverse reinforcement is present, values predicted by ACI 318-77 for development length are conservative.

Since splitting of the concrete cover generally precedes pull-out of the bar for cases where poor confinement is provided, the degree of confinement clearly is a major factor affecting development length. Any predictive equation for development length should, therefore, include confinement parameters.

Untrauer and Warren (26), in their study of stress development of tension reinforcement in beams, concluded that the effect of bar spacing on ultimate bond stress is significant. The smaller the bar spacing used, the lower the ultimate bond stress developed. Thus, the ACI Building Code is overly conservative for widely spaced bars and unsafe for closely spaced bars. Ferguson (27) claimed that if the clear spacing between bars is less than 4 in. or the clear cover is much less than 2 in., the present ACI Building Code development lengths for Grade 60 bars, and possibly Grade 40, are unconservative.

The presence of transverse reinforcement and a compressive stress field are other parameters related to confinement. Transverse reinforcement, such as ties or stirrups, crossing possible concrete splitting surfaces strengthen the critical section after cracking starts by keeping crack widths smaller and slowing crack propagation. In their report, Orangun, Jirsa, and Breen (25) found that the greater the transverse restraint relative to bar diameter, the greater the strength of anchorage over that provided by the concrete cover alone, with an upper limit to this trend.

Jirsa and Marques (28) concluded that the level of axial load in a column did not significantly influence behavior of the hooked bar anchorage of beam end reinforcement. Based on this observation, they recommend that the favorable effects of compressive stress field be disregarded to keep the development length equation on the conservative side.

In the work reported by Orangun, Jirsa and Breen (25), an empirical expression that would include several parameters considered significant was suggested by evaluating available test data. The effect of cover or spacing was reduced to a single parameter. This parameter is the smaller of the clear cover or half the clear spacing between bars. The lowest value of cover or spacing determines the direction splitting will occur.
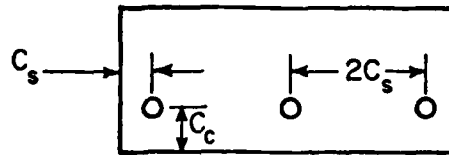
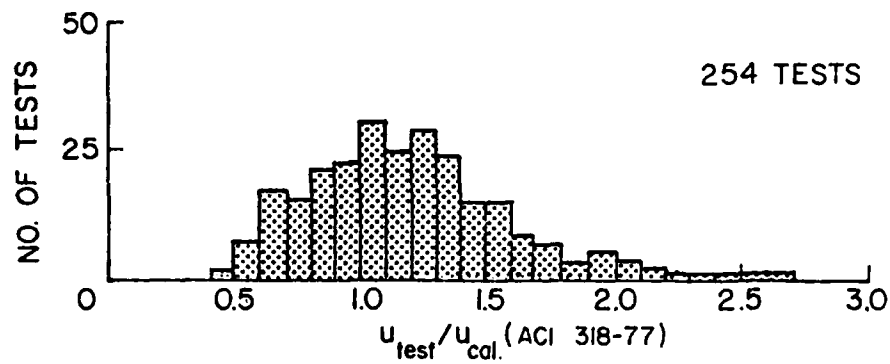Fig. 17   Definition of Cover Parameters

Fig. 18   Comparison of Development Bond Stress Predicted
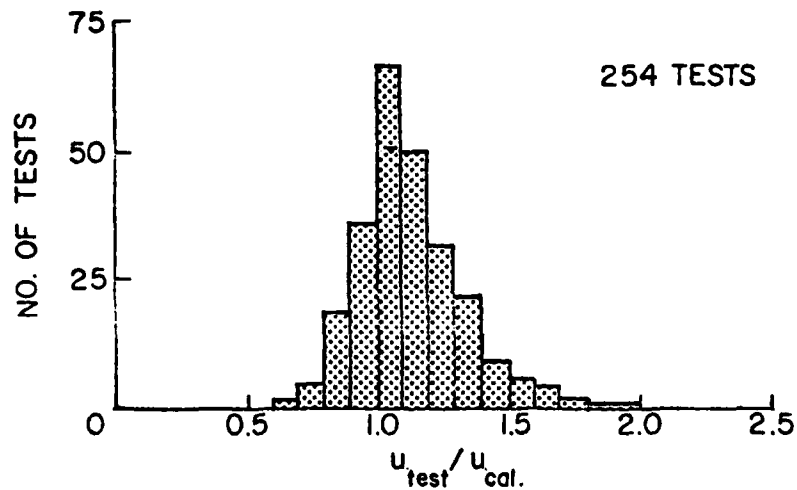by ACI 318-77 and Test Results (from Ref. 31)

Fig. 19   Comparison of Development Bond Stress Predicted
by ACI Committee 408 Equations and Test Results
(from Ref. 31)

Using a regression analysis of a number of test data, the following equation for average bond stress along an anchored bar or a splice was obtained (25):

$$u = \left[1.2 + \frac{3C}{d_b} + \frac{50 \, d_b}{\ell_d} + \frac{A_{tr} f_{yt}}{500 \, s d_b} \right] \sqrt{f_c'} \quad \text{(psi)} \quad (11)$$

where C is the lesser of either C_s or C_c as defined in connection with Eq. 10. Definitions of other parameters appearing above are also as given with Eq. 10. In developing Eq. 11, only tests in which failure in bond occurred prior to yielding of the reinforcement were considered.

Figure 19 shows a histogram of test results on development lengths compared with calculated values based on Eq. 11. The same set of test data used in Fig. 18 was used for comparison. As can be seen, the equation fits a large body of test results very well. Variability is substantially reduced by using Eq. 11. By solving for development length, ℓ_d, and simplifying the resulting expression, Eq. 10, with ℓ_{db} set equal to ℓ_d, is obtained.

Figure 20 shows the required ℓ_{db} for different bar sizes and transverse reinforcement values using the ACI Committee 408 expression, Eq. 10, and ACI 318-77, Eq. 7. In Fig. 20, the distance between concrete surface and center of bars is assumed to be 2.5 in. Clear spacing is equal to 1 in. for a No. 8 bar or smaller, and d_b for a No. 9 bar or larger. For small bar spacings, as assumed in this figure, the proposed development length, ℓ_{db}, is longer compared to ACI 318-77 values. Note also that the significant effect of transverse reinforcement is included in the ACI Committee 408 equation. This is indicated in Fig. 20, which compares a case where transverse reinforcement is provided with one where none is present.

4.3 Bent Bar Anchorage Along Exterior Slab Edges

The development length discussed in the preceding section refers mainly to straight segments of embedded bars, whether at member ends or as they occur in lap splices.

A problem associated with anchorage of slab reinforcement in support regions, besides concrete cover and bar spacing, is the limited space available for anchorage at discontinuous edges. In interior spans of slabs, reinforcement can usually be continued through to adjacent slabs. However, along the exterior edges, there may be difficulty in providing enough space for appropriate anchorage of reinforcing bars. This is especially true when small supporting beams are provided.
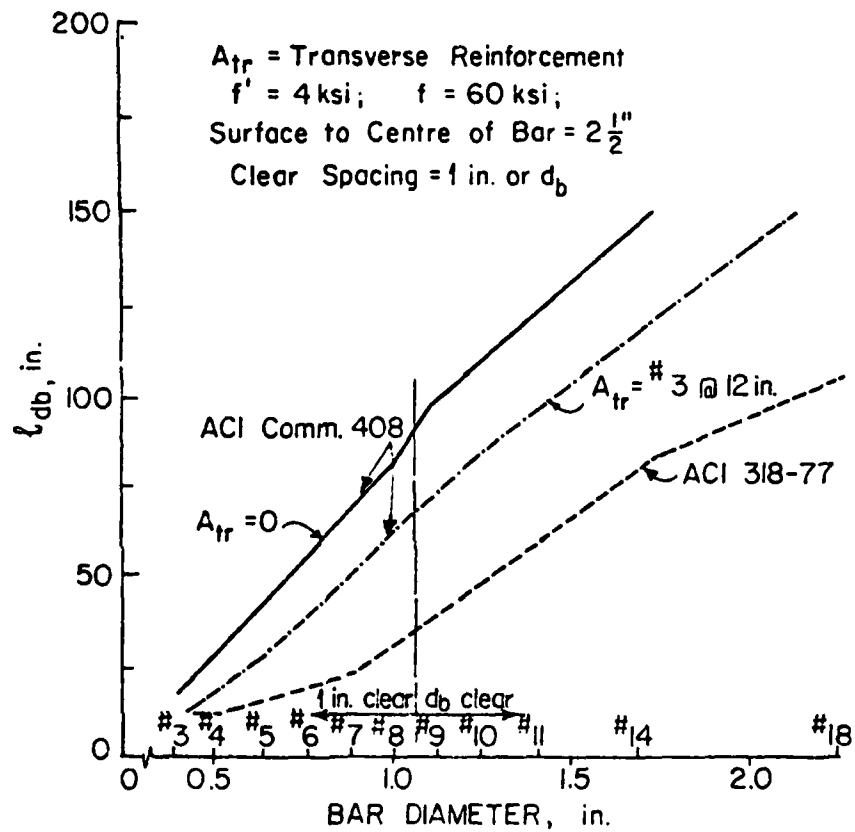
-44-

200

A_{tr} = Transverse Reinforcement
f' = 4 ksi ; f = 60 ksi ;
Surface to Centre of Bar = $2\frac{1}{2}"$
Clear Spacing = 1 in. or d_b

150

ℓ_{db}, in.

ACI Comm. 408

A_{tr} = $^{\#}3$ @ 12 in.

100

A_{tr} = 0

ACI 318-77

50

1 in. clear d_b clear

0

#3 #4 #5 #6 #7 #8 #9 #10 #11   #14   #18

0   0.5   1.0   1.5   2.0

BAR DIAMETER, in.

Fig. 20   Comparison of ACI Committee 408
and ACI 318-77 Development
Lengths (from Ref. 31)

-45-

At discontinuous edges, bent bar anchorage appears to be the most practical alternative if space for the required straight embedment length is not available. Bent bar anchorage consists of a lead section, a curved section, and a tail section, as shown in Fig. 21.

4.3.1 <u>Review of Experimental Work</u>. Minor and Jirsa (32) investigated some factors affecting anchorage capacity of bent deformed reinforcing bars in concrete. A sample test specimen is shown in Fig. 22. Shown in the figure are internal forces acting on a typical beam-column joint core. Force C represents the force on the compressive side of a beam, Force T, the applied force on the bar equivalent to the tension force in the beam, and Forces R_C the reactions within the column. No reinforcement other than the test bars was used within the concrete block.

Results of the tests indicated that a large proportion of the bar force is transferred to the concrete by the lead section preceding the hook. The lead section will be almost at ultimate average bond stress before a significant amount of load is carried by the hook. This is because a large amount of lead end slip is required before the hook develops appreciable load. Also, their test results show that bent bar anchorages are less stiff than straight bar anchorage, and that a bent bar exhibits greater slip than a straight bar of equal bond length.

Based on this investigation, Minor and Jirsa concluded that there is little difference in strength between straight and bent bar anchorages. Also, in terms of reducing slip and maintaining a stiffness of the anchorage comparable to that of a straight bar, 90° hooks are preferable to 180° hooks and the radius of bend should be as large as practicable.

Jirsa and Marques (28) tested twenty-two specimens simulating exterior beam-column joints in a structure, as shown in Fig. 23. The tests were carried out to evaluate capacity of anchored beam reinforcement. Major conclusions drawn from their study are:

(1) Standard hooks conforming to ACI 318-71 (33) embedded in mass concrete can develop stresses well in excess of the yield stress of the bars.

(2) Tail extensions beyond those of a standard hook are ineffective in providing development length since failure results from side splitting of the joint and not by pull-out of the hooked bar.

(3) In general, longer lead embedment lengths result in higher bar stresses at failure. Slip is greater at all stress levels with shorter lead embedment lengths.
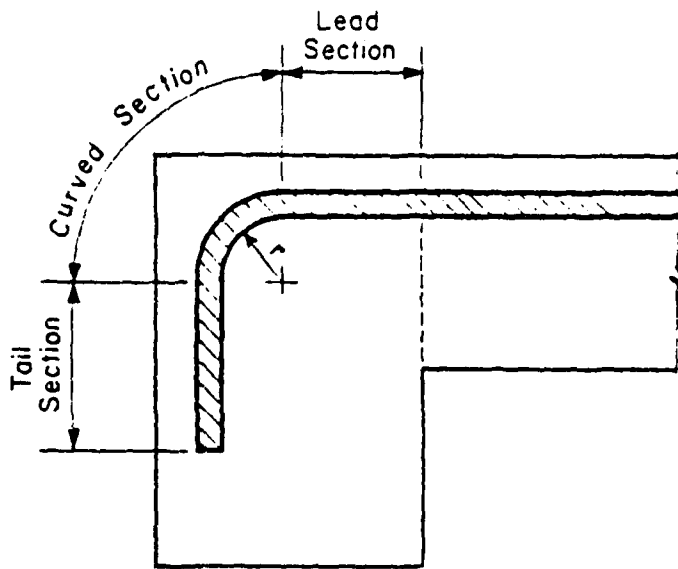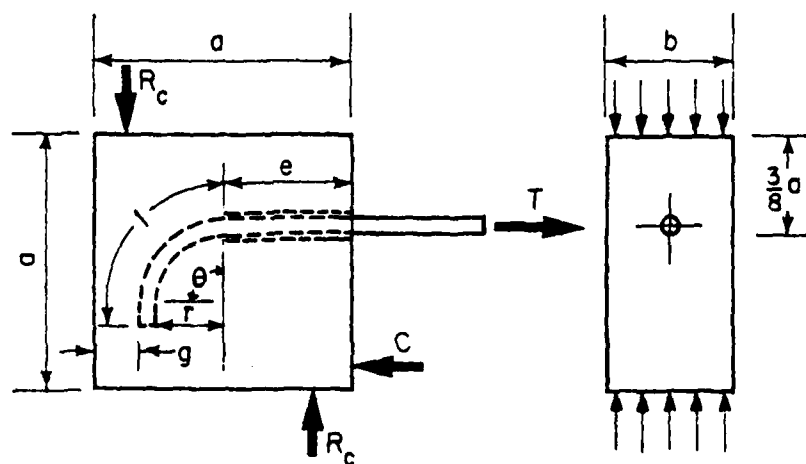
Fig. 21   Hooked Bar Anchorage

$a = 12 \sim 16$ in.; $e = 6 \sim 8$ in.
$b = 8 \sim 12$ in.; $g = 2\frac{7}{8} \sim 4\frac{1}{8}$ in.

Fig. 22   Typical Test Specimen - Minor and Jirsa
(from Ref. 32)

#3 Ties @ 5", Outside Joint

#8 Longitudinal Column Bars

12"x 15" Column

T

C

Height of Column = 50 in.
Assumed Beam Depth = 20 in.

Clear Cover $1\frac{1}{2}$"

ELEVATION

$2\frac{7}{8}$"

SECTION
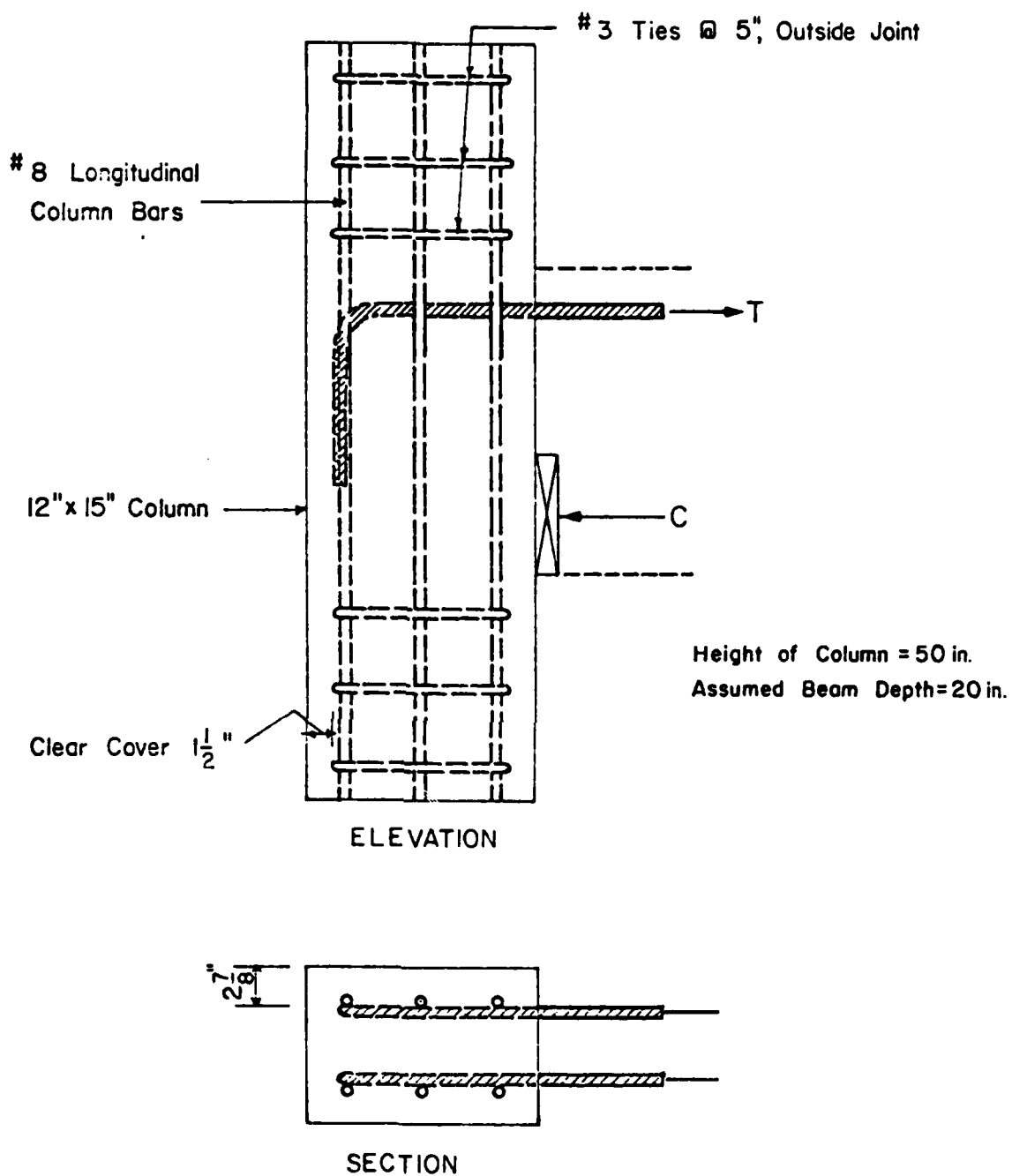
Fig. 23   Test Specimen - Jirsa and Marques
(from Ref. 28)

In an extension of their previous work (28), Pinc, Watkins and Jirsa (29) tested beam-column joints to investigate the influence of straight lead embedment on the strength of hooked bar anchorages. They observed that most of the slip occurs along the straight lead embedment and the curved portion of the hooked bar. Very little slip was measured on the tail extension of the hooks. They combined results of this test program and the results of previous tests, and proposed a simple relationship between embedded length of a hooked bar and anchorage strength. In the relationship, the hook and the straight lead embedment are considered as a unit. Strength of a hooked bar anchorage is treated separately from that for straight bars. The proposed relationship has been incorporated into recommendations by ACI Committee 408 (30) and discussed by Jirsa, Lutz and Gergely (31).

4.3.2 ACI 318-77 Recommendations. The current ACI 318-77 design provisions for hooked bars in tension are a combination of special equations for hooked bars and standard development length provisions as expressed in Eq. 7. In these provisions, tensile stress developed by the hook is expressed as follows:

$$f_h = \xi \sqrt{f_c'} \ (psi) \tag{12}$$

where ξ is a function of bar diameter, yield stress of bars and casting position, and can be selected from a table. The equivalent length of a standard hook, ℓ_e, is obtained by substituting f_h for f_y in the expression for ℓ_d:

$$\ell_e = 0.04 \ A_b f_h / \sqrt{f_c'} \ (in.) \tag{13}$$

The required straight bar development length between the standard hook and the critical section, ℓ', as indicated in Fig. 24, then becomes:

$$\ell' = 0.04 \ A_b (f_y / \sqrt{f_c'} - \xi) \ (in.) \tag{14}$$

Application of Eqs. 12, 13, and 14 is not straightforward because the adjustments in standard hook stress f_h to account for top bars, lightweight concrete, etc., are not clearly defined. In addition, inconsistencies in the values of f_h obtained by using ACI 318-77 have been observed.

4.3.3 ACI Committee 408 Recommendations. In the recommendations of ACI Committee 408 (30), basic development length for Grade 60 hooked reinforcement, ℓ_{hb}, is given by:
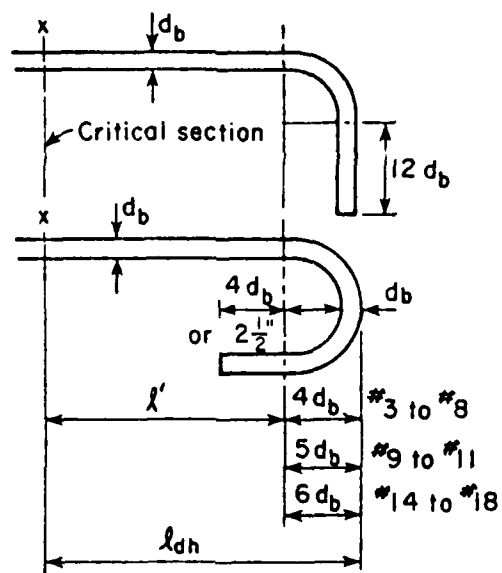
Fig. 24   ACI 318-77 Standard Hook
Geometry

$$\ell_{hb} = \frac{960 \ d_b}{\phi \ \sqrt{f'_c}} \quad \text{(in.)} \tag{15}$$

where ϕ, the strength reduction factor for development length and splices, is assigned a value of 0.8.

Effects of yield stress of reinforcement other than Grade 60, confinement, and lightweight aggregate are accounted for by multiplying Eq. 15 by appropriate factors. The factors are:

    (1)   Reinforcement having
             yield stress other
             than 60,000 psi, $f_y/60{,}000$

    (2)   For #11 bars or smaller
             with side cover normal
             to the plane of the
             hooked bar not less than
             2-1/2 in. and cover on the
             tail extension of 90 degree
             hooks not less than 2 in.,   0.70

    (3)   For additional confinement
             by closed stirrups or hoops
             at a spacing of 3 d_b or less,   0.80

    (4)   Lightweight aggregate replacing
             all or a portion of the aggregate,   1.25

    (5)   Reinforcement in flexural mem-
             bers in excess of that required, $\dfrac{A_{sr}(\text{required})}{A_{sp}(\text{provided})}$

Development length, ℓ_{dh}, of a deformed bar with a hook in tension is considered as a unit, as illustrated in Fig. 24.

In Ref. 29, Pinc, Watkins, and Jirsa provide data that is used as basis for the design approach incorporated in hook provisions proposed by ACI Committee 408. This study indicates that failure of a hooked bar is governed primarily by splitting of the cover parallel to the plane of the hook rather than by pulling out. Also, splitting originates at the inside of the hook where the local stress concentrations are very high. For this reason, Eq. 15 is a function of d_b, which governs the magnitude of compressive stresses on the inside of the hook.

Only standard ACI hooked bars, as indicated in Fig. 24, were considered in tests reported in Ref. 29. Figure 25 shows a comparison, made by Jirsa et al (31), of required length ℓ_{dh}
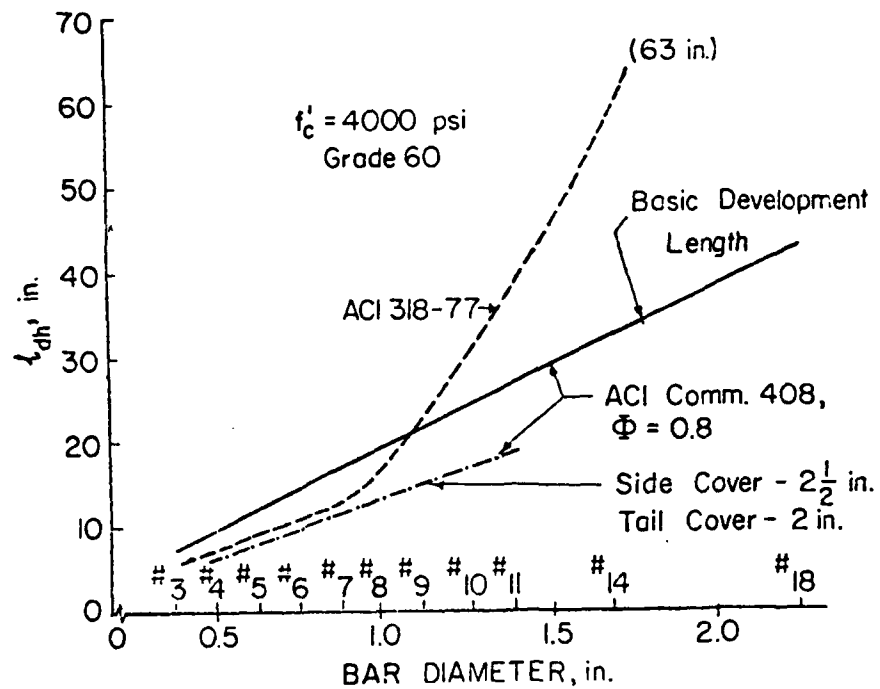
Fig. 25 Comparison of ACI Committee 408
and ACI 318-77 Development
Lengths for Hooked Bars
(from Ref. 31)

using the proposed Eq. 15 and ACI 318-77. The reduction in ℓ_{dh} observed in the new provisions is most pronounced for bars of larger diameter.

4.4 Strength Reduction Factor, ϕ .

In ACI 318-77 no strength reduction factor is specified in the calculation of development length. A strength reduction factor may be thought of as indirectly considered in Eq. 7 for basic development length because of the assumption that the reinforcement develops a stress equal to 1.25 f_y. This is in addition to the ϕ = 0.90 used in all flexural calculations.

Equation 10 for development length and Eq. 15 for hooked bar anchorage in the ACI Committee 408 recommendations are based on developing a stress equal to f_y in the bar. It will be recalled that only those tests in which splitting occurred prior to yielding in the bar were considered in deriving these equations. However, the Committee recommends a strength reduction factor ϕ = 0.80 for use with these equations. Figure 26 shows the distribution of the ratio of test results to computed values using Eqs. 10 and 15 with ϕ = 1.0 and ϕ = 0.80. Also shown are the corresponding curves for ACI 318-77. With ϕ = 0.80, virtually all the test/calculated values for the ACI Committee 408 recommendations lie above a ratio of 1.0.

The strength reduction factor ϕ = 0.80 recommended by ACI Committee 408 has the same effect as the factor 1.25 applied to f_y in the derivation of the ACI 318-77 expression for basic development length, as indicated in Eq. 9. What is important to note is that if ϕ in Eqs. 10 and 15 is viewed as a factor designed to account for variabilities in material properties and structural dimensions, then the anchorage indicated by these equations is intended to develop a maximum stress in the bar equal to f_y only. The procedure used in deriving Eq. 10 and the test data used in validating both Eqs. 10 and 15 in Fig. 26 support this view.

Since, in the tensile membrane action range and prior to rupture of the reinforcement the critical bars are subjected to the ultimate stress, i.e., to stresses generally equal to or greater than 1.25 f_y, there is reason to consider an increase in the anchorage requirements indicated by Eqs. 10 and 15. Until additional information is developed, it is recommended that the value given by Eqs. 10 and 15 for development lengths and hooked bar anchorages be increased by a factor of 1.2 for slabs designed for incipient collapse conditions.
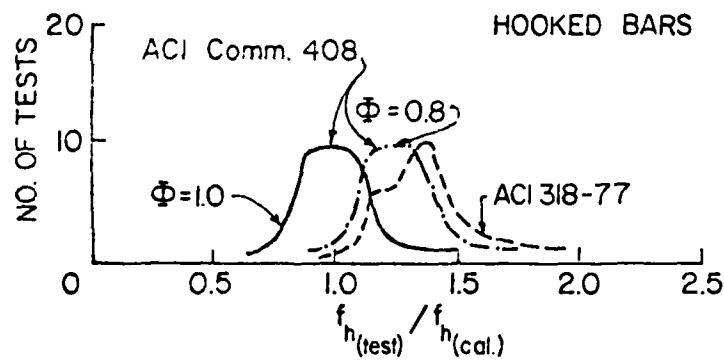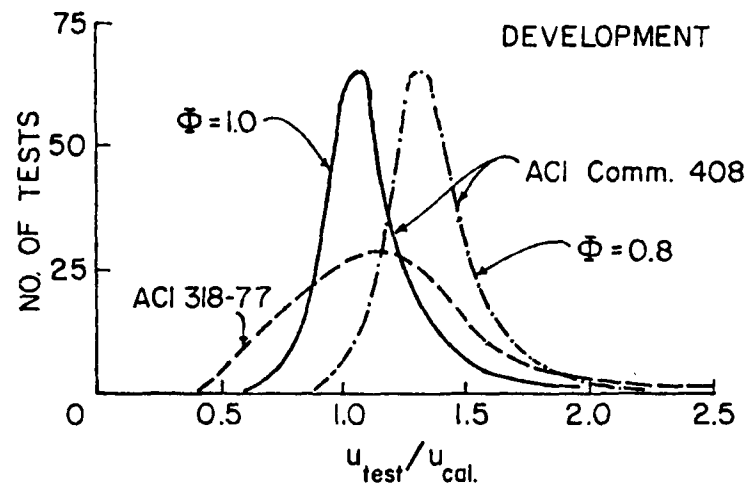
Fig. 26 Effect of Strength Reduction
Factor φ (from Ref. 31)

5. SUMMARY AND RECOMMENDATIONS

This phase of the investigation was originally intended to include a parametric study of variables affecting the behavior of conventionally reinforced concrete slabs in the tensile membrane action range. The objective of such a study would have been to identify the major variables affecting the incipient collapse deflection capacity of slabs subjected to uniformly distributed loading.

The survey of available analytical methods done in Phase I (2) showed that at present there is no available rigorous method for determining the behavior of slabs in the tensile membrane action range. Specifically, no available procedure accounts for the changes in stresses and deformations in different parts of a slab under increasing load.

Because of the lack of a suitable analytical tool needed to carry out a parametric study, resort was made to simplified approaches to the problem of identifying the primary variables. A review of the literature indicated that the best available method of estimating incipient collapse deflection of conventionally reinforced concrete slabs under static uniform loading is given by an expression based on pure membrane action in the slab. An equation based on this assumption was proposed in the Phase I report.

The recommended relationship, i. e.,

$$\delta_{ult} = k \; L_y \; \sqrt{\epsilon_u} \qquad (1)$$

gives the incipient collapse deflection at the center of a slab, δ_{ult}, as a function of two parameters: the short span, L_y, and the rupture strain of the reinforcment, ϵ_u. The proposed expression includes a factor, k, the value of which may be chosen so that the predicted δ_{ult} will have a specified probability of being exceeded.

Two approaches are used in dealing with the uncertainty in the major variables involved in Eq. 1. The first one develops the probability density function for k by considering only measured values of δ_{ult} and ϵ_u reported in slab tests carried to incipient collapse. The second approach assumes k as a function of two random variables, namely, δ_{ult}/L_y and ϵ_u. Probability density functions are constructed for δ_{ult}/L_y from slab test data and for ϵ_u from slab data and coupon test data other than slab data. This second approach allows a more accurate definition of the probability density function for ϵ_u by taking advantage of extensive coupon test data for reinforcing bars.

It is recommended that Eq. 1, with k values indicated in Table 5 be used to calculate the incipient collapse deflection of

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two-way slabs and restrained one-way slabs. The k-values recommended in Table 5 correspond to a probability of approximately 99% that the calculated value of δ_{ult} will exceed in the second approach. In terms of edge rotation, the recommended value of k represents 9° for restrained two-way slabs and 13° for unrestrained, simply-supported, two-way slabs when $\varepsilon_u = 0.15$ is assumed.

The experimental data examined refer only to two-way slabs. No data on incipient collapse deflection is available for restrained one-way slabs. However, it is believed that the use of k-values corresponding to two-way slabs for restrained one-way slabs is conservative.

Also included in the work scheduled for Phase II is development of minimum design and construction requirements to ensure attainment of incipient collapse as defined in Phase I. Since the major parameters affecting incipient collapse deflection were identified and the appropriate relationship proposed in Phase I, the work in Phase II has concentrated on development of minimum design and construction requirements.

In addition to the need for adequate vertical and horizontal supports, the major consideration involved in ensuring development of incipient collapse deflection in slabs is the design of proper anchorage of the reinforcing bars. Experimental data on incipient collapse deflection, as reviewed in Phase I, were obtained by testing small scale specimens. In these specimens, the reinforcement was either welded to stiff boundary elements or anchored in such a way as to eliminate bar pull-out. The anchorage details used in the test specimens are not normally used in practice. Because of this, the tests did not provide useful information on bar anchorage details for incipient collapse.

To develop recommendations on reinforcement anchorage for slabs intended to deform well into the tensile membrane action range, a review of literature on development length for both straight and bent bar anchorages was carried out. Particular importance is given to evaluation of these tests and the resulting anchorage requirements suggested by ACI Committee 408 (30).

In assessing applicability of available data on development length to slabs considered in this investigation, the ranges of selected parameters in the tests were compared with expected ranges of the same parameters in slabs. The particular parameters considered are concrete cover, bar spacing, and depth of concrete below bar.

As indicated in Table 6, the cover thickness considered in the tests on which the ACI Committee 408 recommendations are based represent the upper range of values normally expected in slabs. Furthermore, as discussed in the Section 4.4, the expressions for bar anchorage suggested by the Committee are based on tests

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with a maximum tensile stress in the bar equal to the yield stress, f_y. This contrasts with stresses of the order of 1.25 f_y to 1.40 f_y associated with near-rupture of the reinforcement at incipient collapse. The effect of these differences between the tests used in the ACI Committee 408 recommendations and conditions in slabs at incipient collapse on predicted slab behavior requires experimental verification.

Based on the study conducted and discussed in this report, the following design and construction guidelines are recommended to ensure development of incipient collapse in slabs by tensile rupture of flexural reinforcement:

1. The immediate support system for the slab must be adequate to allow development of tensile membrane action. The surrounding structure or support beams should be capable of providing necessary horizontal and vertical restraints.

    A minimum requirement is that support beams not be involved in the yield-line "collapse mechanism" of the slab. Also, design must ensure that shear failure does not occur at the column supports.

2. Except perhaps for bars close and running parallel to the slab edges, no splicing of reinforcing bars intended to contribute to tensile membrane action should be allowed within the span. Anchorage lengths for bars within supports must be capable of developing full tensile capacity of the reinforcement.

    Positive moment reinforcement should be extended and anchored in the supports by the same amount as required for negative moment steel since both types of reinforcement are subjected to tension in the tensile membrane action stage.

    In case of simply-supported two-way slabs (without edge lateral restraint) the primary flexural reinforcement should be securely hooked around edge bars spanning in the other direction. Adequate concrete cover should be provided for the compression ring portion near the periphery of the slab. The use of diagonally arranged reinforcement may be considered as a means of providing improved bar anchorage for slabs that are square or nearly square in plan.

3. Use of the expressions for development length and hooked bar anchorage suggested by ACI Committee 408 (30), modified by a multiplicative factor of

1.2, is recommended. The current ACI 318-77 (11) requirements can be unconservative for cases where the concrete cover is small, which is typical in slabs.

Because the range of certain dimensional parameters normally expected in slabs differ from those used in the tests on which the Committee 408 recommendations were based, it is strongly recommended that an experimental program be undertaken to check adequacy of anchorage details for slabs loaded to incipient collapse.

4. The amount of slab reinforcement relative to gross section should be sufficient to ensure development of the required tensile membrane strength. A minimum reinforcement ratio of about 1% is suggested to ensure a positive slope in the load-deflection relationship for the tensile membrane action range.

5. Beams of sufficient cross-section and stiffness to satisfy recommendations 1, 2, and 3 should be used along all discontinuous edges of a slab. Beams not only provide a means of anchoring the slab reinforcement but also provide horizontal and vertical support for the plastic membrane.

An experimental program of investigation using large-size specimens is recommended. The purpose of such a program would be to determine accuracy or reliability of the proposed equation for incipient collapse deflection and to verify the applicability of the anchorage requirements suggested by ACI Committee 408 (30) to incipient collapse conditions in slabs.

A brief discussion of the recommended experimental program is given in the Appendix.

ACKNOWLEDGMENT

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APPENDIX A

APPENDIX A
NEED FOR EXPERIMENTAL VERIFICATION

Available experimental data on slabs tested to incipient col-
lapse have provided some basis for identifying the major
variables affecting the incipient collapse deflection capacity
of conventionally reinforced concrete slabs. Test data on de-
velopment length for reinforcing bars in tension have similarly
provided a basis for preliminary recommendations on anchorage
requirements. Both of these groups of data have served to sup-
port the principal recommendations from this investigation.

A number of important questions, however, need further study.
There have been few tests of slabs, whether simply-supported or
restrained, that have been carried out to the point of incipient
collapse. In the case of restrained slabs, the literature sur-
vey indicated only twelve tests by three separate investigators
that were both loaded to incipient collapse and reported values
of the ultimate steel strain, ϵ_u. Reports on six tests of
simply-supported, two-way slabs loaded to incipient collapse
did not include values of the ultimate strain in the reinforce-
ment. A value had to be assumed for this parameter in preparing
the histogram of Fig. 5. All of these slab tests were done
using small scale specimens where the reinforcement was anchored
in such a way as to eliminate all possibility of failure due to
loss of anchorage.

Investigators whose works have been reviewed did not use any
special construction details to obtain large deflections at
incipient collapse. No splicing of reinforcement was used
since the slab specimens were of relatively small size. Most
specimens were singly reinforced and had no confinement rein-
forcement.

As mentioned, and as indicated in Table 6, cover thicknesses
considered in the tests that served as basis for the ACI
Committee 408 recommendations on development length represent
the upper range of values normally expected in slabs. Because
cover thickness is an important factor affecting development
length, adequacy of the Committee 408 recommendations for the
particularly severe conditions associated with incipient col-
lapse requires experimental verification.

It is of considerable interest to determine how well the
equation for incipient collapse deflection developed in this
study can predict the collapse deflection of large-size slab
specimens incorporating practical anchorage details. A check
on adequacy of the ACI Committee 408 recommendations on anchor-
age also appears to be highly desirable.

A test program designed to generate information on the above
questions can be organized most efficiently by considering
problems related to anchorage separately. These tests can use
smaller, less expensive specimens. The primary objective of

such tests will be to determine behavior of slab reinforcement anchorage configurations under conditions simulating incipient collapse. They will also serve to verify the ACI Committee 408 recommendations. Preliminary suggested specimens are shown in Figs. Al and A2.

Once anchorage details have been validated with small specimens, tests on large-size slabs can be designed. The primary purpose of these tests will be to check accuracy of the proposed expression for incipient collapse deflection using large-size specimens. The tests will also serve as proof tests for selected anchorage details to be considered in the first phase of the experimental investigation.

These tests may also be used to investigate the effect of reinforcement arrangement on incipient collapse deflection. Such items as, splicing of bars, location of splices, cut-off of top reinforcement, and the use of single- and double-layered reinforcement may be considered.

A suggested slab test specimen is shown in Fig. A3. The nine-panel slab-beam specimen can be used to evaluate items including splices in edge beams and top reinforcement cut-off.

A single-panel slab specimen with appropriate overhangs may be considered as an alternative to the specimen shown in Fig. A3. This simpler specimen would be less expensive to fabricate.
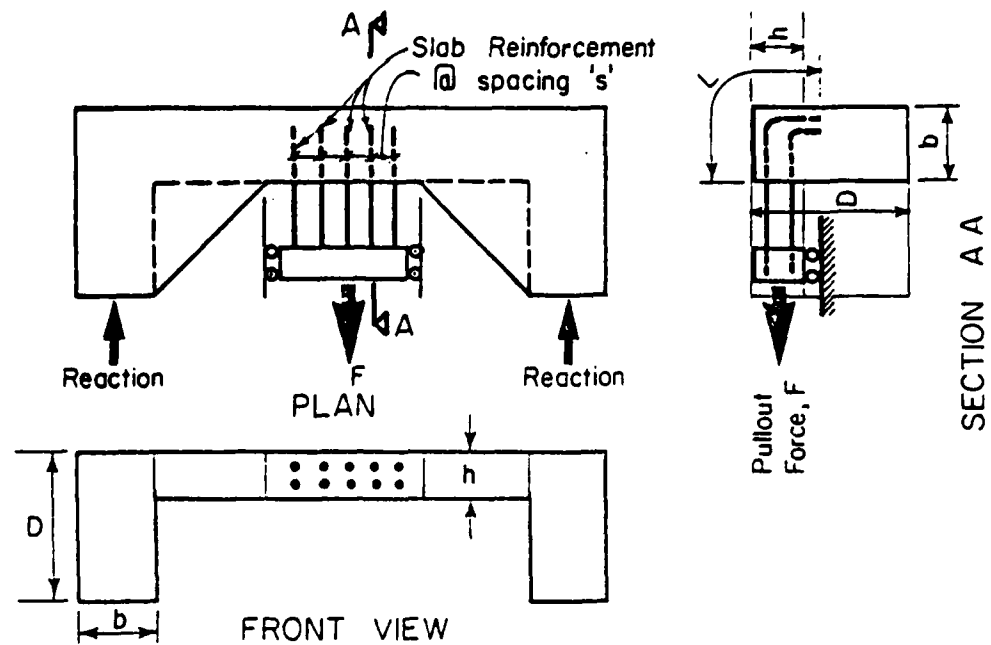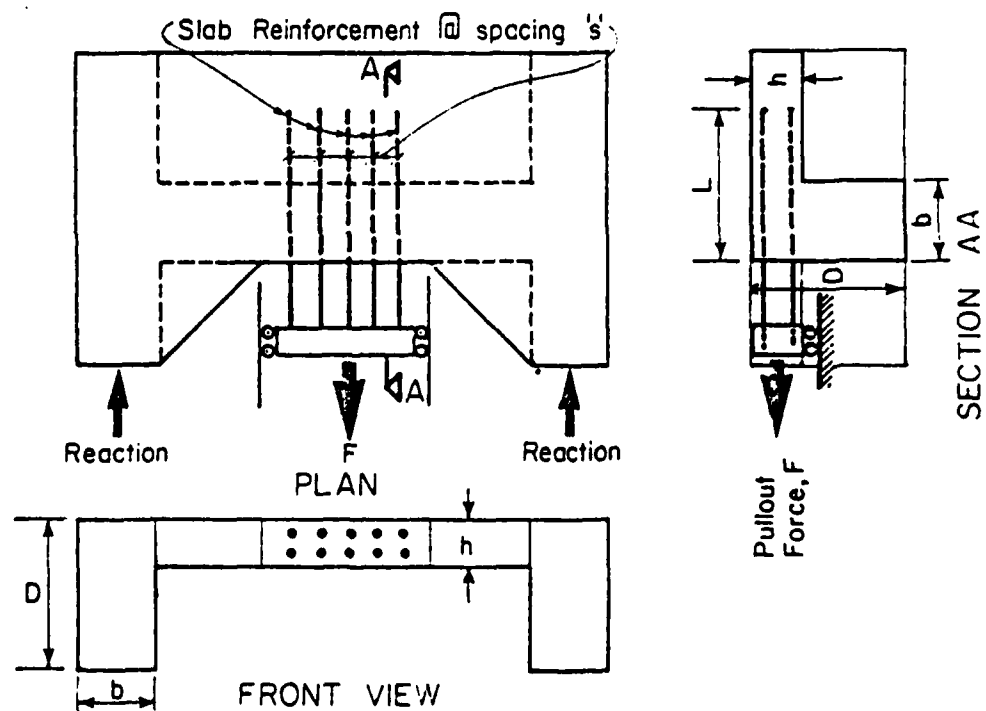
Fig. A1 Element Test Specimen (Anchored in Beam)

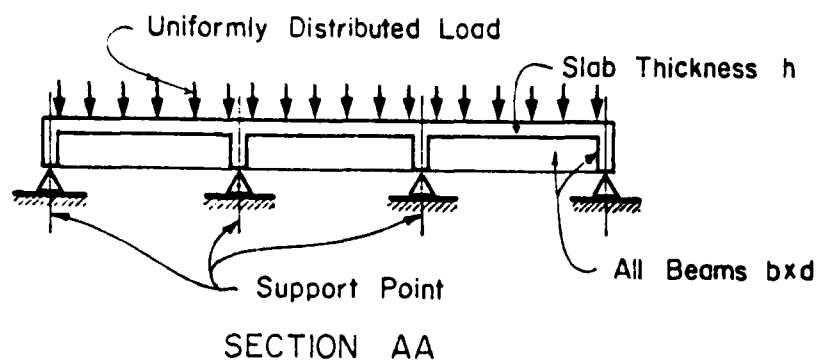Fig. A2 Element Test Specimen (Anchored in Adjacent Slab)
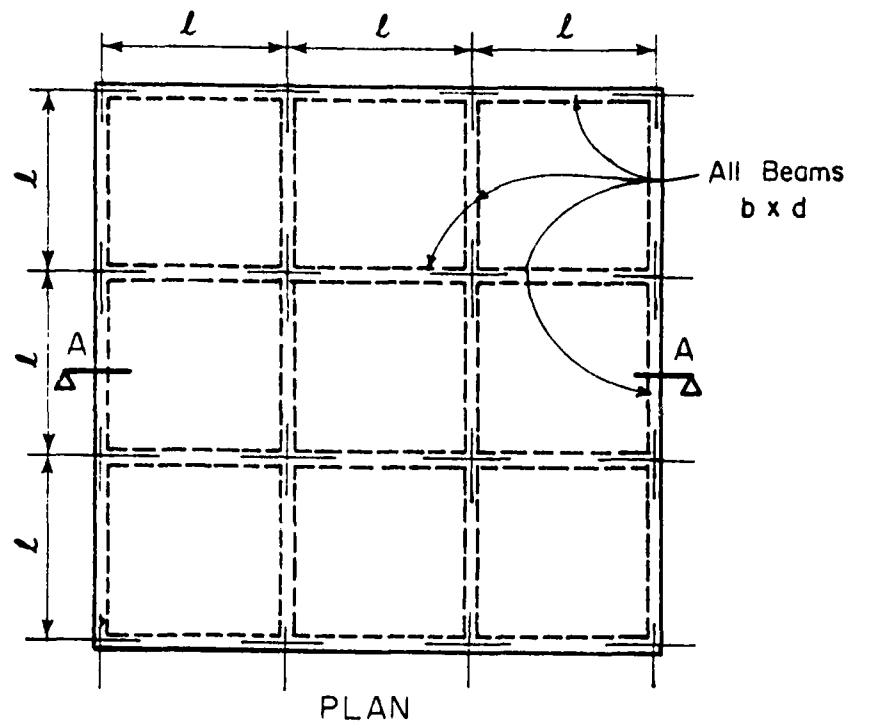
PLAN

SECTION AA

Fig. A3   Nine-Panel Slab Specimen

APPENDIX B

APPENDIX B
ULTIMATE DEFLECTION OF SIMPLY-SUPPORTED,
UNRESTRAINED, ONE-WAY SLABS

In simply-supported one-way slabs, the circumferential com-
pression region or ring that forms along the boundary of
two-way slabs in the tensile membrane range does not occur. In
two-way slabs, this compression ring forms as a result of the
tendency of the outer region of the slab to move inward as
tensile membrane action develops in the central portion of the
slab. Where the tension is in only one direction, as in one-way
slabs, no "self-supporting" compression ring can develop.
Therefore, tensile membrane action cannot be expected in this
type of slab.

The ultimate deflection capacity of simply-supported one-way
slabs is determined by the flexural rotation capacity of the
hinging region or yield line. In uniformly loaded slabs this
yield line occurs at midspan. The rotational capacity of
interest is that associated with the maximum capacity of the
slab, beyond which a rapid decrease in load occurs.

Derivation of an expression for the maximum deflection and cor-
responding edge rotation of uniformly loaded, simply-supported
one-way slabs is presented below. Figure B1 shows the deflected
shape of a slab after the formation of a yield hinge at midspan.
In this analysis, the elastic deformation of the segment of slab
outside the yield hinge is ignored.

The slab will be assumed to be symmetrically reinforced, with
equal amounts of top and bottom reinforcement. At the state of
maximum deflection, prior to collapse, the contribution of com-
pression concrete to the flexural capacity of the hinging region
will be ignored for simplicity.* Under these assumptions, the
neutral axis in the hinging region will be located at mid-depth
of the section, as shown in Fig. B2.

Referring to Fig. B2, the curvature, ϕ, at a cross section
within the hinging region is given by

$$\phi = \frac{2\,\varepsilon_c}{d_c} \quad \text{(1/in.)} \tag{B1}$$

---

*This corresponds to cross section Types II and III discussed in
 Section 5-2 of NAVFAC P-397. Neglecting the contribution of
 the concrete results in a conservative estimate of the flexural
 capacity and a slightly unconservative estimate of curvature.

where:

d_c = distance between the centroids of compression and tension reinforcement, in.

Total rotation at the hinge, θ_h, is obtained by integrating ϕ over the length of the hinging region,

$$\theta_h = 2 \int_0^{\frac{\ell_h}{2}} \phi \, dx \qquad \text{(rad.)} \qquad \text{(B2)}$$

where:

ℓ_h = length of hinging region, in.

x = horizontal distance from center of span, as shown in Fig. B3.

The hinging region is defined here as the portion of the slab where the moment is greater than the yield moment of the slab section.

Under a uniformly distributed load, the bending moment diagram is defined by a second order equation as follows:

$$M = \frac{wx^2}{2} + \frac{wL^2}{8} \qquad \text{(lbs-in.)} \qquad \text{(B3)}$$

where:

w = uniformly distributed load, lbs/in.
L = span of slab, in.

If the elastic portion of curvature is ignored, the curvature distribution within the hinging region can also be expressed by a second order equation. The second order equation is derived as follows.

The curvature at any section within the hinging region, ϕ, is given by:

$$\phi = \frac{M_y}{(EI)_e} + \frac{M - M_y}{(EI)_p} \qquad \text{(1/in.)} \qquad \text{(B4)}$$
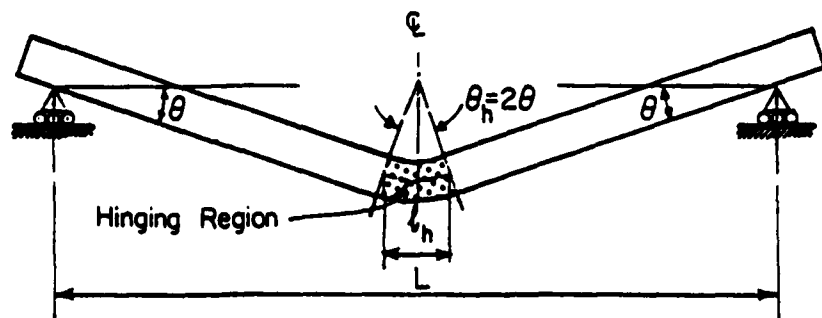
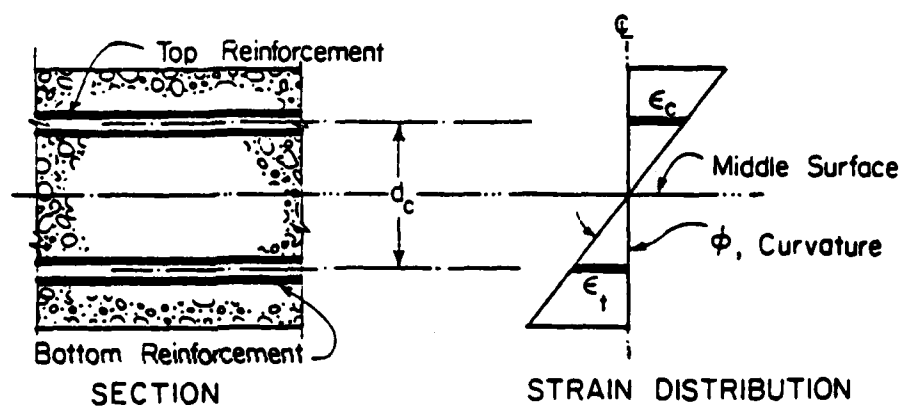Fig. Bl   Collapse Mechanism of Simply-Supported One-Way Slab

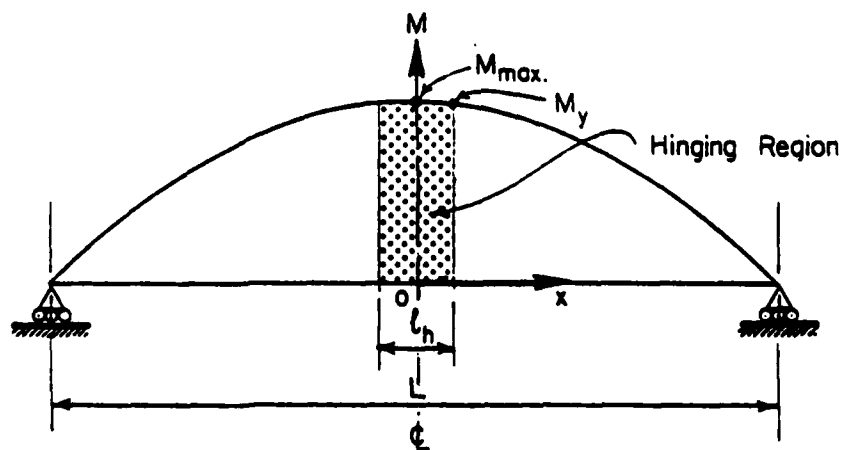Fig. B2   Strain Distribution at a Cross Section in Hinging Region

Fig. B3   Moment Diagram

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$$- \frac{\ell_h}{2} \leq x \leq \frac{\ell_h}{2}$$

where:

M = moment at a section where curvature is to be calculated, lbs-in.

M_y = yield moment, lbs-in.

$(EI)_e$ = elastic flexural rigidity, lbs-in.^{2}

$(EI)_p$ = post-yield flexural rigidity, lbs-in.^{2}

The first term on the right-hand side of Eq. B4, representing the elastic component of curvature, is generally small in magnitude compared to the second term and can therefore be neglected. By dropping this term and substituting the value of M from Eq. B3 into the remaining expression in Eq. B4, one obtains

$$\phi = \frac{1}{(EI)_p} \left(- \frac{wx^2}{2} + \frac{wL^2}{8} - M_y \right) \qquad (1/in.) \qquad (B5)$$

$$- \frac{\ell_h}{2} \leq x \leq \frac{\ell_h}{2}$$

With the elastic component of curvature assumed as negligible, the curvature at $x = \ell_h/2$ where the hinging region ends, is zero. Evaluation of Eq. B5 at $x = \ell_h/2$ gives

$$- \frac{w}{2} \left(\frac{\ell_h}{2} \right)^2 + \frac{wL^2}{8} - M_y = 0 \qquad (B6)$$

The condition of maximum deflection in the slab is assumed as determined by a postulated maximum usable concrete strain in compression. Beyond this value of strain the concrete in the hinging region is assumed to have lost effective contact with the compression reinforcement and to contribute nothing to the flexural capacity of the section. The limiting condition is thus characterized by the flexural capacity of the hinging region being provided only by the tension and compression steel. In the absence of lacing, the shear capacity of the hinging region is reduced drastically and the compression reinforcement can buckle as it loses support from the concrete. This condition is followed by a rapid loss of load capacity in the slab.

If the maximum usable strain in the concrete at the level of the compression reinforcement is denoted by ε_{max}, the following relationship can be established for the section at midspan, using Eq. B1:

$$\epsilon_{max} = \frac{d_c}{2\,(EI)_p}\left(\frac{wL^2}{8} - M_y\right) \qquad (B7)$$

From Eqs. B2 and B5, the cumulative rotation over the hinging region, θ_h, can be expressed as follows:

$$\theta_h = \frac{2}{(EI)_p}\int_0^{\frac{\ell_h}{2}}\left(-\frac{wx^2}{2} + \frac{wL^2}{8} - M_y\right)dx$$

$$= \frac{\ell_h}{(EI)_p}\left(-\frac{w\ell_h^2}{24} + \frac{wL^2}{8} - M_y\right) \quad (rad.) \qquad (B8)$$

Eliminating w and ℓ_h from Eq. B8 by utilizing Eqs. B6 and B7, yields the following expression for the total hinge rotation, θ_h,

$$\theta_h = \frac{4\,\epsilon_{max}\,L}{3\,d_c}\sqrt{1 + \frac{1}{\dfrac{M_y\,d_c}{2(EI)_p\,\epsilon_{max}}}} \qquad (rad.) \qquad (B9)$$

Yield moment, M_y, and post-yield flexural rigidity, $(EI)_p$, can be expressed as follows:

$$M_y = f_y\,A_s\,d_c \qquad (lbs-in.) \qquad (B10)$$

$$(EI)_p = A_s\,E_y\left(\frac{d_c}{2}\right)^2 \times 2 \qquad (lbs-in.) \qquad (B11)$$

where:

f_y = yield stress of the reinforcement, psi.
A_s = area of the reinforcement, in.^{2}
E_y = post-yield slope of the stress-strain
        relationship for the reinforcement, psi

By substituting M_y and $(EI)_p$ from Eqs. B10 and B11 into Eq. B9, and dividing the resulting expression by 2, one obtains an expression for the edge rotation of the slab,

$$\theta_{ult} = \frac{\theta_h}{2} = \frac{2\epsilon_{max}}{3}\frac{L}{d_c}\sqrt{1 + \frac{1}{\frac{f_y}{E_y\,\epsilon_{max}}}} \qquad \text{(rad.)} \qquad \text{(B12)}$$

The ultimate deflection, δ_{ult}, for simply-supported one-way slabs is given by,

$$\delta_{ult} = \theta_{ult}\left(\frac{L}{2}\right) = \frac{\epsilon_{max}}{3}\frac{L^2}{d_c}\sqrt{1 + \frac{1}{\frac{f_y}{E_y\,\epsilon_{max}}}} \qquad \text{(in.)} \qquad \text{(B13)}$$

It will be noted that the ultimate deflection as derived above is a function of a number of parameters, namely, the maximum usable compression strain in concrete, span length, distance between top and bottom reinforcement, yield stress of reinforcement and post-yield slope of the stress-strain relationship of reinforcement.

The application of Eqs. B12 and B13 is illustrated below for particular combinations of the relevant variables.

Example 1:

Given: $\epsilon_{max} = 0.010$

$d_c = 12$ in.
$L = 180$ in.
$f_y = 60,000$ psi
$E_y = 200,000$ psi

Maximum edge rotation,

$$\theta_{ult} = \frac{2 \times 0.010 \times 180}{3 \times 12}\sqrt{1 + \frac{1}{\frac{60}{200 \times 0.010}}} = 0.018 \text{ rad.}$$
$$\text{or } 1.0 \text{ degrees}$$

$$\delta_{ult} = 0.018\left(\frac{L}{2}\right) = 0.009L$$

Example 2:

Given: Same values as in Example 1 except $\epsilon_{max} = 0.015$.

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Maximum edge rotation,

$$\theta_{ult} = \frac{2 \times 0.015 \times 180}{3 \times 12} \sqrt{1 + \frac{1}{\frac{60}{200 \times 0.015}}} = 0.033 \text{ rad. or } 1.9 \text{ degrees}$$

$$\delta_{ult} = 0.033 \left(\frac{L}{2}\right) = 0.016L$$